Hybrid platforms and bargaining power*

Martin Stancsics†

August 21, 2024

Latest version: stanmart.github.io/phd-thesis/application.pdf

Abstract

This paper explores the effects of a platform selling their own products in addition to acting as intermediaries (hybrid platform) in a setting where bargaining takes place between the platform and the potential entrants. It highlights a so-far underappreciated aspect of hybrid platforms: having their own products increases their bargaining power over the other sellers. This change in bargaining might lead to lower entry, and thus reduced product variety which in turn can have negative welfare consequences for the consumers. The results and methods described in this paper are also applicable to other similar settings, such as vertical integration with a monopolistic upstream supplier.

1 Introduction

Although market structures resembling various aspects of what we call platforms have existed for some time, the term gained widespread usage only in the 2000s, with the advent of digitization and a number of behemoth technology companies playing matchmakers on the Internet. Since then, the concept of a platform has become prominent in the business world, drawn increasing regulatory attention, and become a topic of intense academic interest. Platforms have several unique features that differentiate them from more traditional market structures. The most prominent of those are multi-sidedness and network effects, which most of the early literature (for an overview, see Rochet and Tirole 2006) and policy debate (e.g. Fletcher et al. 2021; Calvano and Polo 2021) has focused on. However, in recent years, another important and potentially concerning aspect has been getting more attention: the hybrid operation of certain platforms. Such platforms act as intermediaries (deciding the rules) and market participants (competing with other entrants) at the same time.

While hybrid platforms are not an entirely new phenomenon, they are becoming increasingly common in various industries. Perhaps the most prominent example is Amazon, which sells its own products while simultaneously hosting a large number of third-party sellers. However,

*I would like to thank my advisors, Armin Schmutzler and Marek Pycia, for their numerous thoughtful comments and suggestions on this paper. Without their guidance, this paper would look very different. I am also grateful to all participants of the Swiss IO Day, as well as the Topics in Microeconomic Theory and Market Design seminars at the University of Zurich, for the lively discussion and helpful feedback.

†University of Zurich, martin.stancsics@econ.uzh.ch

examples abound in other industries, as well. For example, each of the largest digital distribution platforms for computer and smartphone applications (Google Play, Apple App Store, and Microsoft Store) sells its own applications in addition to third-party offerings. In the video game industry, platforms owning studios and publishing games under their own brand is the standard. Many video streaming services (e.g. Netflix, Amazon Prime Video, Hulu) offer a mix of their own and licensed content. One can find examples of similar behavior outside the platform setting, too. For example, several car manufacturers are planning to sell directly to consumers in addition to selling through their dealership networks.

As deciding the rules of the game and competing in it at the same time gives rise to some obvious concerns, policymakers have been paying increasingly close attention to these platforms in recent years. Various pieces of legislation have been proposed to regulate or even ban e-commerce platforms from selling their own products (Phartiyal 2019; Reynolds 2022; Council of European Union 2022). Furthermore, one of the most prominent recent antitrust cases, Microsoft acquiring the video game publisher Activision/Blizzard for 69 billion USD, is also a case of a platform becoming increasingly hybrid. This merger not only increased market concentration on the publishing side of the video game industry, but also had implications for the competition between Microsoft as a platform owner and the other game publishers. The deal was under investigation by multiple antitrust authorities, including the US Federal Trade Commission, the European Commission, and the UK Competition and Markets Authority (Livni and Merced 2023).

Parallel to the increasing regulatory interest, the academic literature on hybrid platforms has also been steadily growing. Hagiu, Teh, and Wright (2022) investigate how practices, such as self-preferencing and steering, can distort competition on hybrid platforms. They argue that, while such practices are problematic and can have negative welfare effects, with proper regulation the platform selling its own products can be beneficial for consumers. In contrast to this positive result, Anderson and Bedre-Defolie (2021) find that even in the absence of such behavior, allowing hybrid operation might have negative consequences due to platforms' incentive to exclude competitors from the market. Gutierrez (2021) shows that general conclusions are hard to draw, and welfare effects are platform and product-specific. For example, using a mechanism design approach, Kang and Muir (2022) demonstrates that welfare consequences greatly depend on whether the platform faces competition in the upstream market.

This paper aims to contribute to this discussion by examining an important but underexplored aspect of platforms: the bargaining power disparity between the latter and the other market participants, and how this disparity is affected by the platforms' hybrid operation. I propose an analytically tractable that captures many important aspects of bargaining between one large and a continuum of small players: namely, a hybrid platform facing a continuum of potential entrants. In this paper's distortion-free setting with lump-sum entry fees, hybrid platforms are not detrimental to consumer welfare under the usual assumption of the platform setting the entry fee unilaterally. I show that, in contrast, in the presence of bargaining, hybrid operation can reduce consumer welfare. The intuition is that hybrid operation increases the platform's bargaining power against the entrant sellers. This, in turn, leads to a higher entry fee and fewer entrants, resulting in lower consumer surplus. These observations constitute a so far overlooked aspect of hybrid platforms that policymakers should be aware of. Due to this model's generality, these results are applicable not only to hybrid platforms, but also to many other settings, such as upstream producers having their own downstream outlets or retailers with private labels.

Many of the results are not tied to the specifics of the model but are more general. They do not depend on a specific demand structure, and there is even some flexibility with regard to the assumptions about bargaining outcomes. I show that, even in such a stylized setting, some interesting observations can be made about the effects of the hybrid operation. In particular, when the platform's and the potential entrants' products are substitutes, increasing the platform's product variety will lead to a decrease in the number of entrants. I also give sufficient conditions for this decrease in entry being so large as to cause a decrease in total profits.

The closest model to the one proposed in this paper is Anderson and Bedre-Defolie (2021). The key difference between the two boils down to how the entry fee is determined. In Anderson and Bedre-Defolie (2021), the platform sets a percentage entry fee, and the entrants decide whether or not to enter. In contrast to this, I assume that (1) platforms charge a lump-sum entry fee and that (2) this entry fee is the result of a negotiation process[1] between the platform and the entrants. The second point is a novelty not only compared to Anderson and Bedre-Defolie (2021), but also to the existing literature on hybrid platforms. Furthermore, in contrast to Hagiu, Teh, and Wright (2022), this model is free from any distortive behavior, such as self-preferencing or steering, and the negative results are purely driven by changes in the platform's bargaining position.

The set of more general results in this paper goes beyond the aforementioned studies in the sense that they illuminate the effects of hybrid operation as a more abstract phenomenon, and thus also highlight the similarities between the hybrid platform literature and adjacent ones. Certain structures, such as vertical relationships or even traditional retail stores, share several features with platforms (notably, the presence of a dominant, indispensable entity) and can be modeled in much the same way. Therefore, the methodology and results in this paper are also of interest to researchers studying questions such as retailers having their own private labels (Steiner 2004) or vertical integration in upstream-downstream markets (O. Hart, Tirole, et al. 1990; Aghion, Griffith, and Howitt 2006). De Fontenay and Gans (2005) and Montez (2007), in which bargaining between the upstream and downstream firms take center stage, are particularly closely related. Among empirical studies in related settings, Ho and Lee (2017) looks at the effect of insurer competition on health care prices, while Crawford et al. (2018) explores the effects of vertical integration on television markets.

This paper is also related to various other strands of the industrial organization literature. Most generally, it is a contribution to the research agenda on understanding platforms and their role in the economy (e.g. Rochet and Tirole 2003; Hagiu 2004; Armstrong 2006; Evans et al. 2011; Lee 2014). Furthermore, it is somewhat adjacent to the literature on the importance of exclusive content (e.g. Hagiu and Lee 2011; Lee 2013; Dou 2014; Weeds 2016), with the difference that instead of exclusive content giving an advantage against competing platforms, in this paper, own products provide an advantage over potential entrants. More generally, many results and

[1] The bargaining process is modeled using a solution concept from cooperative game theory, namely the Shapley value. This allows for a tractable analysis while at the same time capturing many of the essential features of bargaining. Examples of this approach in the industrial organization literature include Montez (2007), as well as O. Hart and Moore (1990), Levy and Shapley (1997), Inderst and Wey (2003) and Brügemann, Gautier, and Menzio (2019) among others.

concepts of this paper are also applicable outside the context of (multi-sided) platforms.

Finally, this paper also belongs to the relatively small set of models that use concepts from cooperative game theory in an industrial organization setting. Such examples include the aforementioned Montez (2007), as well as O. Hart and Moore (1990), Levy and Shapley (1997), Inderst and Wey (2003) and Brügemann, Gautier, and Menzio (2019), among others. In contrast to the majority of those, which focus on games with a finite number of players, I utilize an oceanic game (a continuum of small players instead of a finite number), demonstrating that this can considerably simplify the analysis in certain cases. Therefore, the modeling approach presented in this paper may also be useful in other settings.

The rest of the paper is organized as follows. In Section 2, I introduce two model of hybrid platforms: a benchmark with the platform setting an entry fee unilaterally, and the main model, where the platform and the fringe firms engage in bargaining. After that, Section 3 demonstrates that the two have very different implications for consumer welfare. Appendix A then examines the effect of different assumptions about the bargaining process. Finally, Section 4 summarizes the results and discusses possible future work.

2 Model

This section introduces the model used throughout the paper. I use the terms "platform" and "fringe sellers" and present my ideas in the context of an intermediated market, such as an online marketplace or an application store. However, the model is quite general, and the results apply to a broader class of settings, such as vertical markets where the upstream firm can sell directly to consumers or retail stores with their own private labels. Furthermore, while the discussion in the main text focuses on a specific demand structure and profit sharing rule, most of the results apply to a broader class of models. Appendix B explores the generalizability of these results, highlighting some common themes in settings with hybrid behavior.

Assume that there are two types of players in the market: a platform P, a continuum of fringe sellers F_i $(i \in \mathbb{R}^+)$, and a continuum of consumers C_j $(j \in [0,1])$. The fringe firms have one product each, which they can only sell through the platform. Without the platform, they make zero profits. In addition to acting as an intermediary between the fringe and the consumers, the platform itself may also produce and sell a number of products directly to the consumers. If it does, it is referred to as a hybrid platform.

The main distinguishing feature of this model is that instead of assuming that entry fees or royalties are set by the platform and that the fringe treats them as take-it-or-leave-it offers, I assume that the platform and the fringe engage in some kind of bargaining over their total profits. The rest of this section describes the bargaining rule and other details of the model and is structured as follows. Section 2.1 provides an overview of the model's stages, the timing of the game, as well as the equilibrium concept used in this paper. Then, Section 2.2 formally describes each stage of the game. Finally, Section 2.3 discusses some of the assumptions made in the model, and their implications.

2.1 Overview and timing

I consider two versions of the model: a benchmark case with the platform setting the entry fee unilaterally, and a bargaining model where the platform and the fringe firms negotiate over the entry fee. Each version has four main stages corresponding to determining four sets of endogenous variables: (1) the platform (and the fringe) setting (negotiating) the entry fee, (2) the fringe firms deciding whether to enter the market, (3) the platform and the fringe setting the prices of their products, and (4) consumers making their purchase decisions. The main difference between them lies in how the entry fee is determined and, correspondingly, the order of the first two stages.

The timing of both models is illustrated in Figure 1. The square brackets indicate the endogenous variables decided at that stage of the game. In the benchmark model (Figure 1a), the game starts with the platform announcing and committing to an entry fee K_F ($\mathbf{T_1}$). Next, each fringe firm decides whether to create a product at cost I_F and enter the market. ($\mathbf{T_2}$) After that, the platform and the fringe firms simultaneously choose the prices for their products, engaging in monopolistic competition ($\mathbf{T_3}$). Finally, consumers make their purchase decisions and profits are realized ($\mathbf{T_3}$).

The bargaining model has slightly different timing, as shown in Figure 1b. The platform cannot commit to an entry fee at the beginning, so the game starts with the fringe firms deciding whether to invest in creating a product at cost I_F ($\mathbf{T_1}$). After that, the entry fee is decided as a result of some negotiation[2] between the platform and the firms that have made the investment ($\mathbf{T_2}$). Then, the firms that invested pay the platform entry fees, too. From this point on, the game proceeds in the same way as in the benchmark model: the platform and the fringe firms set their prices ($\mathbf{T_3}$), and consumers make their purchase decisions ($\mathbf{T_4}$).

The main difference between the benchmark and the bargaining model lies in determining the entry fee K_F. In the former, the platform unilaterally sets the fee, and the fringe treats it as a take-it-or-leave-it offer. On the other hand, in the bargaining model, I assume that the entry fee results from negotiation between the parties. More precisely, in the latter case, I assume that the platform's and the fringe's total profits at the end of the game correspond to their Shapley values. This, in turn, uniquely determines the entry fee. Thus, the negotiation at time ($\mathbf{T_2}$) can equivalently be conceptualized as bargaining over the final profits, as well.

For both models, this structure corresponds to a perfect-information extensive-form game. Consequently, the solution concept I use is the subgame perfect equilibrium. As the platform's product variety (N_P) is assumed to be exogenous, the only strategic variables are (1) the fringe players' entry decisions, (2) the platform's entry fee (only in the benchmark model), (3) the prices of the products and (4) the consumers' purchase decisions.

The subgame perfect can be characterized in these two specific games as follows.

Definition 1. *An equilibrium of the game is a tuple* $(N_F, K_F, p_{P_i}, p_{F_i}, x_{P_i}, x_{F_i})$ *such that:*

1. *The fringe entrants make zero total profits (including the investment cost* I_F *and the entry fee* K_F*).*

2. *The platform's entry fee* K_F *is such that*

[2] The negotiation process, or rather its outcome, is described in detail in Appendix C.2.

(**T₁**) Platform unilaterally sets the entry fee
$[K_F]$
↓
(**T₂**) Potential entrants decide to enter at cost $I_F + K_F$
$[N_F]$
↓
(**T₃**) Platform and fringe set product prices
$[p_{P_i}, p_{F_i}]$
↓
(**T₄**) Consumers make consumption decisions
$[x_{P_i}, x_{F_i}]$

(a) Benchmark model (platform sets the entry fee unilaterally)

(**T₁**) Potential entrants decide to invest at cost I_F
$[N_F]$
↓
(**T₂**) Entry fees are negotiated between platform and N_F fringe firms
$[K_F]$
↓
(**T₃**) Platform and fringe set product prices
$[\pi^v_{Fi}, \pi^v_P]$
↓
(**T₄**) Consumers make consumption decisions
$[\pi^t_{Fi}, \pi^t_P]$

(b) Bargaining model (platform and fringe negotiate over the entry fee)

Figure 1. Timing of the models

benchmark case: *they maximize total platform profits;*

bargaining case: *total platform profits are equal to the Shapley value of the platform.*

3. *Product prices p_{F_i} and p_{P_i} are such that they maximize per-product profits, given the products on the market ($P_i, i \in [0, N_P]$ and $F_i, i \in [0, N_F]$).*

4. *x_{F_i} and x_{P_i} are the outcome of consumers maximizing their utility, given the prices.*

2.2 Details

2.2.1 Demand

Let us now describe each stage of the game in more detail, starting with the final two subgames, as they are identical in both models. This part of the model is based on Anderson and Bedre-Defolie (2021), who utilize the exact same utility structure as the one described here. Imagine that there is a unit mass of consumers looking to buy one product each. They choose from a continuum of products, which are either produced by the fringe (indexed by F_i), by the platform (P_i) or are amongst a unit mass of outside options (0_i). Customer j derives the following utility from buying product T_i[3]:

$$u^j_{T_i} = v_{T_i} - p_{T_i} + \mu\epsilon^j_{T_i} \quad \text{for } T_i \in \{F, P\},$$

where v_{T_i} is the value of product T_i, p_{T_i} is its price, and $\epsilon^j_{T_i}$ is an idiosyncratic taste shock. Throughout this section, I assume that the value of each fringe product is the same, and the

[3] To avoid duplication, I denote the different types of products by T_i, where $T \in \{F, P, 0\}$ (fringe products, platform products, and outside options, respectively). This notation is used throughout the paper for other variables having fringe and platform counterparts, as well.

same goes for the platform products. I.e., $v_{T_i} = v_T \, \forall \, i$. This is to simplify the analysis but is not crucial for the results.

$\epsilon^j_{T_i}$ is assumed to be independent and identically distributed (i.i.d.) across consumers and products and follow a standardized Type I Extreme Value distribution. This distributional assumption, along with the fact that each consumer consumes only one product, will lead to a tractable, logit-form demand function. The demand for product T_i, which arises from consumers maximizing their utility, is denoted by x_{T_i}.

2.2.2 Production

Each (horizontally differentiated) product is produced by a single, monopolistically competitive (fringe) seller. The production entails a constant marginal cost c_{T_i}. As with the value, I assume that the marginal cost is the same for all products: $c_{T_i} = c_T$. Facing the demand described in the previous paragraphs, the sellers choose their price p_{T_i} to maximize profits. The price of the outside option is normalized to zero.

2.2.3 Fringe entry and platform fees

Now, let us turn to stages 1 and 2 of the game: the determination of the entry fee and the fringe firms' entry decision. I start with the benchmark model, where the platform sets the entry fee unilaterally. Then, I compare and contrast this with the bargaining model, where the entry fee results from a negotiation between the platform and the fringe firms.

Benchmark case I assume that there is a continuum of potential fringe entrants, indexed by $i \in \mathbb{R}^+_0$. Each can create a product at an exogenous investment cost I_F. If they decide not to, they make zero profits and do not participate in the game's later stages. Furthermore, these firms can only sell their products through the platform, for which the platform can charge a lump-sum entry fee K_F. The timing of these decisions is as follows: (1) the platform sets and commits to an entry fee K_F, (2) the fringe firms decide whether to enter the market and pay investment cost I_F and entry fee K_F.

Bargaining case In contrast to the benchmark model, in this case, the platform cannot commit to an entry fee: it is decided as a result of a negotiation between the platform and the fringe firms. In order to keep the model tractable, I model this in a reduced-form manner: I assume that the platform and the entrants agree on an entry fee K_F, which makes the total profits net entry fees equal to the Shapley value of the players. Due to this assumption, the timing of the model is also different. First, the fringe firms decide whether to invest in creating a product at cost I_F. After this, the platform and the firms that have made the investment negotiate the entry fee. Finally, the firms that entered pay the entry fee to the platform.

More specifically, the resulting entry fees look as follows. Assume that N_F fringe firms choose to invest in creating a product in the first stage of the game. Now, for every sN_F ($s \in [0, 1]$), calculate the total profits that would be realized if the platform and n_f fringe firms were the only players in the market, and they engaged in monopolistic competition as described in the previous paragraph. Let us denote these profits by $\pi_F^v(N_P, sN_F)$ and $\pi_P^v(N_P, sN_F)$, and the corresponding industry-wide total profits by Π and $\Pi(N_P, sN_F)$.[4]

Proposition 1. *The Shapley value of the platform and the fringe firms, and thus final total profits, are given by:*

$$\pi_P^t(N_P, N_F) = \int_0^1 \Pi(N_P, sN_F)\mathrm{d}s,$$
$$\pi_F^t(N_P, N_F) = \int_0^1 \partial_2 N_F \Pi(N_P, sN_F)\mathrm{d}s.$$

Proof of Proposition 1. The cooperative game is described in detail in Appendix C.2. This result is a direct consequence of Proposition C.1. □

2.3 Discussion

Let us now discuss some of the non-standard assumptions made in the model and their implications. I look at four main categories in this section: the timing of the game, the costs of entry for fringe firms, the way the monopolistic competition is modeled, and the way the bargaining process is modeled.

2.3.1 Costs of entry

For each potential fringe entrant F_i, entering the market has two separate costs: an exogenous investment cost I_F and the lump-sum platform entry fee K_F. The first one, I_F, can be conceptualized as usual fixed costs, such as the cost of setting up production or designing a product. Without it, fringe firms would always make positive profits in the bargaining model, and therefore, the free-entry equilibrium would not exist. Furthermore, it is quite a reasonable assumption in most settings, as some of the costs of entering the market are sunk and not paid to the platform.

Meanwhile, the second cost, K_F, is a payment to the platform for using its services. While percentage fees are more common both in the literature and in practice, I assume a lump-sum fee for a few reasons. First, and most importantly, revenue-based fees enter the entrants' pricing decisions, and thus create a channel through which platforms distort competition. I want to show that hybrid platforms can significantly impact competition even in the absence of such distortions. Furthermore, due to this distortion, the resulting game could not be represented as a transferable utility game, and the proposed solution concept would not be applicable. Finally, lump-sum fees do make sense in settings when individual negotiation takes place and contracts are not standardized.

2.3.2 Timing of the game

The two models are designed to be as similar as possible, also with respect to the timing of the game. The only difference is in the order of the first two stages, which is necessary due to the

[4] Throughout this paper, I use the notation π_T to denote the profits realized by player(s) T, and Π to denote the total industry profits. Furthermore, π_T^v denotes profits from sales (i.e., profits not including investment costs and entry fees), whereas π_T^t stands for total profits.

nature of the two different entry fee-setting mechanisms.

In the case of the benchmark model, if the platform set entry fees after fringe firms make their investment decisions, the platform would charge an entry fee that would make the fringe profits zero without taking into account the investment cost I_F. This would lead to negative fringe profits and, thus, to a trivial equilibrium where no firm enters. Therefore, the entry fee setting must come before the investment decisions of the fringe firms.

Contrary to this, in the bargaining model, the opposite order is necessary. This is because the entry fee is the result of a negotiation, and by the time the negotiation takes place, it must already be clear which firms are engaging in it. Otherwise, the bargaining process (either in its cooperative, reduced-form version or in the non-cooperative, extensive form one) would not be well defined. Therefore, the fringe firm's entry decisions[5] must come before the determination of the entry fee.

2.3.3 Product pricing

Due to their infinitesimal market share, the fringe sellers' pricing decisions are rather straightforward: they do not have to take into account the effect of their prices on the aggregate demand. This is typically not the case for the platform, as it can affect a non-zero measure of the prices. Therefore, when setting the price for product P_i, it would optimally like to take into account how it affects demand on its other products P_j $(j \neq i)$. Despite this, I assume that the platform prices its products as if they were produced by separate, monopolistically competitive sellers.

I do this for a number of reasons. For one, it simplifies the analysis without affecting the main results qualitatively. More importantly, it lets me focus on the main question of this paper: how the platform's hybrid operation affects bargaining power. If the platform priced its products strategically, there would be an additional channel through which its product variety could affect the outcomes, as, in contrast to the fringe firms, it takes into account the effect of its prices on the aggregate demand. This would make it harder to disentangle the effects of hybrid operation from the effects of strategic pricing. This assumption can then be seen as a best-case scenario for consumers and the fringe, in which the platform is given as little market power as possible.

A number of other papers using the same demand system (e.g. Anderson, Erkal, and Piccinin 2020; Anderson and Bedre-Defolie 2021) achieve essentially the same outcome by relying on the timing of the model. Specifically, if the platform sets product prices before the fringe firms make their entry decisions, the aggregate demand will not depend on the platform's product prices[6]. This, in turn, means that the platform will optimally price its products as if they were produced by separate sellers. Such timing would be problematic in the current paper because the aim is to keep the benchmark and bargaining models as similar as possible. It would be unrealistic to assume that the platform can commit to prices before the fringe firms make their investment decisions, while at the same time, it cannot commit to an entry fee in the latter model.

Finally, even taken at face value, this assumption about the platform's pricing is quite reasonable in certain settings. For example, the platform might produce its products through many legally separate subsidiaries, thus having no control over operational decisions but still

[5]With respect to creating a product. Entering the platform and paying the entry fee are separate decisions, which take place during the negotiation.

[6]At least in the hybrid regime. The optimum in the pure retailer regime is somewhat different.

earning profits from their sales. Alternatively, even within a single firm, there might be some internal competition between the different product teams, leading to a similar outcome.

2.3.4 Bargaining over the platform entry fee

Although the main reason for utilizing cooperative game theory in this model is tractability, it is by no means the only one. This way of modeling bargaining outcomes, while uncommon, has been used to great effect in several other papers in the industrial organization literature (e.g. Montez 2007; O. Hart and Moore 1990; Levy and Shapley 1997; Inderst and Wey 2003; Brügemann, Gautier, and Menzio 2019). Furthermore, it has a number of appealing properties. For example, it is closely related to the marginal contributions of the players to the total value, and, as shown in Appendix B.4, it is also quite intuitive in terms of comparative statics. Furthermore, and perhaps more importantly, there are many ways to place it on non-cooperative microfoundations by setting up non-cooperative games for which the Shapley value is a subgame perfect equilibrium (e.g. Gul 1989; S. Hart and Mas-Colell 1996; Stole and Zwiebel 1996b). Appendix C.1 provides an example of how this specific model can be placed on such microfoundations. The subgame perfect equilibrium of the fully non-cooperative model described in that section coincides with the equilibrium of the bargaining model in the main text. Therefore, one can think of the cooperative game as a more tractable representation of the one from Appendix C.1.

Let us also discuss the specifics of the cooperative game in which the Shapley value is calculated. For the Shapley value (or any other cooperative solution concept) to be well-defined, the corresponding coalitional game must also be precisely specified. Appendix C.2 describes the latter in detail. In short, the idea is the following: given any subset of the players (coalition), players can predict total, industry-wide profits, as the pricing subgame has a unique subgame perfect equilibrium for any subset of the players. This total profit is taken as the value of that coalition, thus defining a characteristic function. Then, according to the definition of the Shapley value, every firm (including the platform) will get their average marginal contribution to this industry-wide profit (with the average taken over all possible orderings of the firms).

A straightforward extension of this bargaining process would be to use the more general concept of random order values (Weber 1988). Stancsics (2024) examines this solution concept for a similar game but in a more abstract setting. Appendix C.2.1 also takes a step in this direction by introducing bargaining weights and utilizing the weighted value.

3 Results

I will now present the model's main results, starting from the final subgame and working backward. I also demonstrate that the equilibrium exists and is unique in each subgame, and therefore, the complete game also has a unique equilibrium. The propositions in this section are generally proven within a more general framework, and those presented here are special cases. The reason for emphasizing the more special case is twofold. First, the more concrete model is better for expositional purposes and is a good fit for the context of platforms. Second, the link between consumer surplus and the number of products is straightforward under the logit assumption, making obtaining welfare results possible.

For this reason, the proofs are presented together with the more general model in Appendix B. Most of the results only rely on Assumptions B.1 to B.6, and do not require the specific, logit-like demand function described previously. Therefore, I proceed to prove those results in two steps. First, I demonstrate that the model satisfied these assumptions. Second, in Appendix B, I prove the corresponding results for the more general case.

3.1 Demand and producer profits

As the final, price-setting subgame is the same in both the benchmark and the bargaining model, the results in this subsection apply to both. Discrete choice models with type I Extreme Value errors give rise to a logit-type demand function (e.g. Small and Rosen 1981). More specifically, Anderson and Bedre-Defolie (2021) shows that it gives rise to the following demand function.

Proposition 2. *The demand for product i of producer $T \in \{P, F\}$ is given by:*

$$x_{T_i} = \frac{\exp\left(\frac{v_T - p_{T_i}}{\mu}\right)}{A}$$

where

$$A = \int_0^{N_F} \exp\left(\frac{v_F - p_{F_i}}{\mu}\right) \mathrm{d}i + \int_0^{N_P} \exp\left(\frac{v_P - p_{P_i}}{\mu}\right) \mathrm{d}i + 1. \tag{1}$$

Proof of Proposition 2. See Theorem 1 in Anderson and Bedre-Defolie (2021). □

Let us call $v_T - p_{Ti}$ the net value of product i. As one would expect, demand is increasing in this net value and decreasing in the competitors' net values. Furthermore, demand for each product is increasing in μ, which describes the degree of product differentiation or the importance of taste shocks. Finally, as each producer is infinitesimal, its pricing decision does not affect the aggregate A. This last property makes the optimal prices and profits of the producers very simple, as shown in the next proposition.

Proposition 3. *The profit maximizing price for product T_i is*

$$p^*_{T_i} = c_T + \mu,$$

and the profit from selling that product is

$$\pi^{v*}_{T_i} = \mu \frac{\exp\left(\frac{v_T - c_T - \mu}{\mu}\right)}{A}. \tag{2}$$

For ease of notation, let us define the following:

$$V_T = \exp\left(\frac{v_T - c_T - \mu}{\mu}\right).$$

V_T can be thought of as the value of the product, also accounting for marginal costs and taste heterogeneity. In this logit demand system, the primitive parameters v_T and c_T only affect the outcomes through this value. In fact, equilibrium per-product demand and variable profit can simply be expressed as V_T/A and $\mu V_T/A$, respectively, where

$$A = N_P V_P + N_F V_F + 1 \tag{3}$$

denotes the total aggregate. Therefore, to simplify the notation, I will use V_T and A instead of the more cumbersome expressions in the rest of the paper.

Finally, another key feature of this demand system is that under optimal pricing, consumer welfare only depends on the size of the aggregate (Anderson, Erkal, and Piccinin 2020). In particular, consumer surplus is proportional to the logarithm of the aggregate: $CS = \mu \log(A)$. This fact makes welfare analysis relatively simple in this setting.

3.2 Benchmark: platform sets entry fee unilaterally

Now, let us examine entry fees and fringe entry decisions in the benchmark model. Recall that there is an infinity of potential fringe entrants looking to enter the market. Therefore, total profits in equilibrium must be zero. Combined with the profit function, this gives the following expressions for the equilibrium number of fringe firms and the equilibrium size of the aggregate.

Proposition 4. *If entry costs I_F and K_F are low enough, the equilibrium size of the aggregate is*

$$A = \mu \frac{V_F}{K_F + I_F}. \tag{4}$$

and the equilibrium number of fringe firms is

$$N_F = \frac{\mu}{K_F + I_F} - N_P \frac{V_P}{V_F} - \frac{1}{V_F}. \tag{5}$$

Otherwise, $N_F = 0$ and the equilibrium size of the aggregate is

$$A = N_P V_P + 1.$$

Note that in the first case (Equation (4)), the size of the aggregate does not depend on the platforms' product variety or the platform's product value. The intuition behind this is that per-firm fringe profits only depend on these factors indirectly, through the size of the aggregate. Therefore, the zero profit condition for the fringe firms pins down the aggregate in equilibrium (as long as the fringe is feasible). That is, an increase in N_P will replace some fringe entrants, but the free entry condition will pin down the same aggregate regardless of the number of platform products.

Now, let us turn to the optimal entry fee set by the platform and its total profits (consisting of revenue from its own sales and the collected entry fees). The following proposition establishes these both for the hybrid and the pure retail regimes, and Figures 4b and 5 demonstrates them graphically.

Theorem 1. *The optimal entry fee when the platform is operating in the hybrid regime is unique* *and given by*

$$K_F^{opt} = \sqrt{\mu I_F V_F} - I_F. \tag{6}$$

The platform's total profit in this case is

$$\pi_P^t = \mu - 2\sqrt{\frac{I_F \mu}{V_F}} + \frac{I_F}{V_F}(N_P V_P + 1). \tag{7}$$

When the optimal mode of operation is retail, the platform's profit is

$$\pi_P^t = \pi_P^v = \mu \frac{N_P V_P}{N_P V_P + 1}. \tag{8}$$

First, let us look at the case of the platform finding it optimal to operate in the hybrid regime. As illustrated by Figure 4b, the optimal entry fee (Equation (6)) does not depend on either the platform's product value or product variety. This is because due to the lump-sum nature of the entry fee, the platform can extract all the surplus from the fringe firms, and thus chooses K_F to maximize total industry profits. The latter is a function of the aggregate minus the investment costs of the fringe firms. As I show later in Theorem 2, the aggregate is independent of the platform's product variety in the hybrid regime. Therefore, the optimal entry fee is also independent of it.

On the other hand, Figure 5[7] shows the platform's profit (Equations (7) and (8)) is increasing in the number of its products.

Corollary 1.

$$N_F(N_P) > 0 \implies \frac{\mathrm{d}\pi_P^t}{\mathrm{d}N_P} = \frac{V_P}{V_F} I_F.$$

This is a rather mechanical result: the platform's product variety is exogenous, and possible investment costs are not modeled. If the platform is operating in the hybrid regime, fewer fringe firms are needed to achieve the (fixed) equilibrium aggregate, so less is spent on (essentially wasted) investment costs. Consequently, in the hybrid regime, the derivative of optimal profits with respect to the platform's product variety is simply the fringe's investment cost, adjusted for the possible difference in product value.

Now let us examine consumer welfare as a function of the platform's product variety[8]. Under the assumed demand system, consumer surplus is proportional to the logarithm of the aggregate. Therefore, one only needs to understand how fringe entry, and thus the aggregate, changes with the platform's product variety to understand the latter's effect on consumer welfare. The following theorem establishes results for these three variables in the benchmark model.

Theorem 2. *In the benchmark model, if $N_F(N_P) > 0$ (hybrid regime), then the following hold.*

[7] $\frac{\mathrm{d}X}{\mathrm{d}N_P}$ denotes equilibrium comparative statics. I.e., if the variable X is a function of N_P and N_F, then

$$\frac{\mathrm{d}X}{\mathrm{d}N_P} = \frac{\partial X(N_P, N_F(N_P))}{\partial N_P}$$

.

[8] The comparative statics for the platform's product value, V_P, are similar.

- *The equilibrium number of fringe firms is decreasing in the platform's product variety:* $\frac{\mathrm{d}N_F}{\mathrm{d}N_P} = -\frac{V_P}{V_F} < 0$.
- *The equilibrium size of the aggregate and consumer surplus are independent of the platform's product variety:* $\frac{\mathrm{d}A}{\mathrm{d}N_P} = \frac{\mathrm{d}CS}{\mathrm{d}N_P} = 0$.

If $N_F(N_P) = 0$ *(pure retailer regime), then both the aggregate and consumer surplus are increasing in the platform's product variety:*

- $\frac{\mathrm{d}A}{\mathrm{d}N_P} = V_P > 0$,
- $\frac{\mathrm{d}CS}{\mathrm{d}N_P} = \frac{\mu V_P}{(1+N_P V_P)^2} > 0$.

The most important, and perhaps surprising, result of this theorem is that the platform's product variety does not affect consumer welfare in the hybrid regime. This is because the optimal entry fee, which is independent of N_P, pins down the aggregate. As shown in Figure 4a, an increase in the platform's product variety will simply replace fringe firms in a constant ratio, such that the aggregate remains constant (Figure 6a). This, in turn, implies that consumer surplus is also unaffected by the platform's product variety in the hybrid regime, as demonstrated in Figure 6b.

When the platform finds it optimal to operate as a pure retailer, its own products are the only products on the market. Therefore, quite mechanically, the aggregate, and thus consumer surplus, are increasing in the number of the platform's products (unshaded parts of Figure 6).

Finally, note that the platform's profit under the hybrid regime is higher than under the retail regime whenever the former is feasible. Therefore, for a given N_P, the platform prefers to operate in hybrid mode and does not want to exclude fringe firms from the market. This fact, together with the Theorem 2, implies that the platform's product variety always has a weakly positive effect on consumer welfare.[9]

3.3 Bargaining: platform and entrants negotiate over entry fees

Let us now turn to the main contribution of this paper: the case where the platform and the fringe firms negotiate over the division of profits. As the platform cannot choose and commit to an entry fee, the first two periods are switched compared to the benchmark model. The game starts with fringe firms' investment decisions, after which, in the second period, bargaining takes place between the platform and the entrants to determine the entry fee.

As described in Section 2.2.3, participants negotiate over the aggregate profits that they expect to obtain in the subsequent period (assuming the same non-collusive, monopolistic pricing as before). From Equation (2), the total profit achieved by the platform and N_F fringe firms is given by

$$\Pi(N_P, N_F) = \mu \frac{N_F V_F + N_P V_P}{N_F V_F + N_P V_P + 1}. \tag{9}$$

[9]This is in contrast to Anderson and Bedre-Defolie (2021), where a platform operating in hybrid mode sets higher royalties to create a price advantage for its own products. The reason for this difference lies in the type of entry fee: they assume a revenue-based, proportional fee, which distorts prices. In contrast, I assume a non-distortive lump sum fee in this paper.

Bargaining outcomes are determined according to Shapley values, given by Proposition 1. Based on these, closed-form expressions for platform and fringe profit shares for this class of demand systems can be obtained.

Proposition 5. *The platform's total profits are*

$$\pi_P^t = \mu \left[1 - \frac{\log\left(1 + \frac{N_F V_F}{N_P V_P + 1}\right)}{N_F V_F} \right].$$

The total profits of the whole fringe (excluding investment costs) are given by

$$\pi_F^t = \mu \left[\frac{\log\left(1 + \frac{N_F V_F}{N_P V_P + 1}\right)}{N_F V_F} - \frac{1}{N_P V_P + N_F V_F + 1} \right].$$

Let us establish a couple of partial equilibrium results about the profit shares before moving on to the main results. Consider the number of fringe firms N_F as fixed. Then, total profits, as given by Equation (9), are clearly increasing in the number of platform products. The next proposition shows that the platform's profits are also increasing in N_P, while the fringe's profits are decreasing.

Proposition 6. *For any $N_F > 0$, the platform's profits are increasing in the number of its products, while the fringe's profits are decreasing:*

$$\frac{\partial \pi_P^t}{\partial N_P} > 0, \quad \frac{\partial \pi_F^t}{\partial N_P} < 0.$$

This is a rather striking result. Even though the total size of the pie increases, the slice that the fringe can obtain decreases. This happens because of the substitutability of the fringe's and the platform's products[10]: as the latter increases its product variety, the marginal benefit of adding each fringe firm decreases, leading to a deterioration in the fringe's bargaining position.

Let us now endogenize N_F and turn to this section's main results: how the platform's dual-mode operation affects product variety and consumer welfare. I start by showing that, like in the benchmark model, the equilibrium is unique in this case, as well. This, and many other results in this section, rely on the following lemma.

Lemma 1. *For any $N_P, N_F \geq 0$, the fringe profit function is either concave or decreasing in N_F:*

$$\frac{\partial \pi_F^t(N_P, N_F)}{\partial N_F} < 0 \text{ or } \frac{\partial^2 \pi_F^t(N_P, N_F)}{\partial N_F^2} < 0 \quad \forall N_P, N_F \geq 0.$$

This lemma states that the fringe profit function is hump-shaped in the sense that it starts with an increasing[11], concave part, after which it may turn into a decreasing (but not necessarily

[10] Proposition B.4 demonstrates that the direction of the change depends on the cross derivative of the total profit function.

[11] More precisely, while the lemma does not explicitly state that the function is increasing for low values of N_F, the fact that $\pi_F^t(N_P, 0) = 0$ and $\pi_F^t(N_P, N_F) > 0 \forall N_F > 0$ implies it.

concave or convex) function of N_F. Figure 2 illustrates this property.

While this lemma's primary purpose is to help establish the results about fringe entry and consumer surplus for the bargaining case, it is also interesting in its own right. It shows that the number of fringe firms does not necessarily have a monotonic effect on the total profits of the fringe. That is, even though industry-wide profits are increasing in N_F, the amount that the fringe can obtain may decrease after a certain point, not only as a fraction of the total but also in absolute terms. The underlying reason is that as the number of fringe firms increases, their bargaining power decreases due to their substitutability with each other. This decrease can be large enough to offset the increase in the size of the total pie.

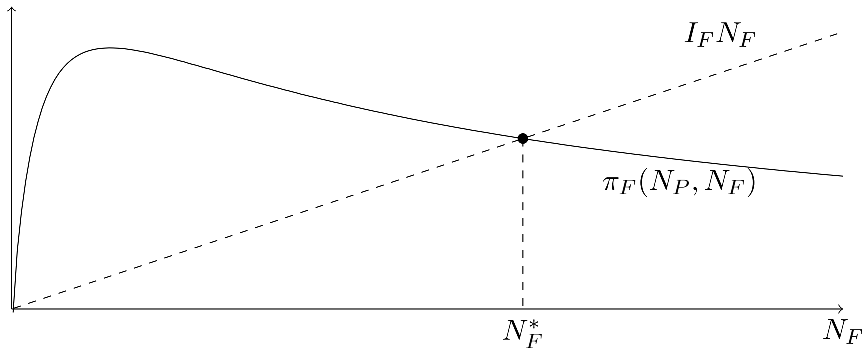


Figure 2. An example equilibrium. Lemma 1 states that the fringe profit function is concave or hump-shaped in N_F, guaranteeing at most one intersection with the linear entry cost function.

An almost immediate corollary is that such a function must have (apart from the trivial $N_F = 0$ case) at most one crossing with total investment costs, a linear function of N_F. Let us denote this point as N_F^*. Furthermore, if such a crossing exists, then it must be that $\pi_F^t(N_F) > I_F N_F \; \forall \, N_F < N_F^*$, and $\pi_F^t(N_F) < I_F N_F \; \forall \, N_F > N_F^*$ (see Figure 2 for an illustration of this idea). Therefore, N_F^* must be the unique number of equilibrium entrants. The following proposition formalizes this observation.

Proposition 7. *The equilibrium number of entrants, N_F^*, is unique in the bargaining case.*

Now, let us examine how the platform's product variety affects fringe profits and the equilibrium number of fringe firms. First, let us establish that fringe profits are decreasing in the number of the platform's products. Note that this is a partial equilibrium result: N_F is assumed to be fixed, while the platform's product variety is increased.

Proposition 8. *For any $N_P, N_F \geq 0$, the fringe profit function is decreasing in N_P:*

$$\frac{\partial \pi_F^t(N_P, N_F)}{\partial N_P} < 0.$$

As illustrated in Figure 3, together with the hump-shaped fringe profit function, Proposition 8 implies that an increase in the platform's product variety leads to a reduction in the number of fringe firms. What is not immediately apparent is how this affects the total size of the aggregate and consumer welfare. In the benchmark case, the decrease in the number of fringe firms was exactly offset by an increase in the platform's product variety, and the aggregate remained

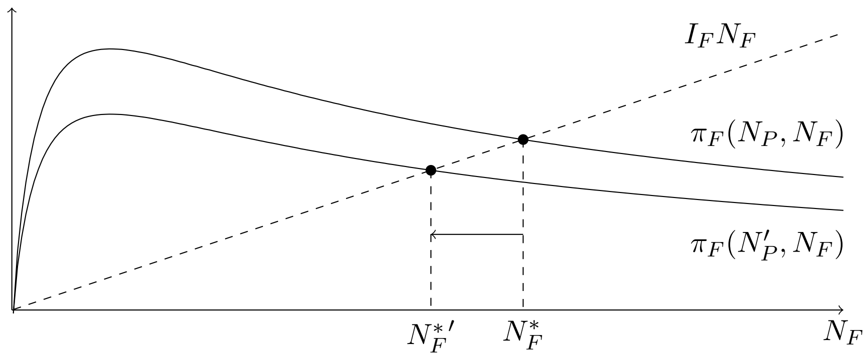


Figure 3. Illustration of comparative statics for equilibrium entry. If the change in N_P decreases the fringe's profits for every $N_F \geq 0$, then equilibrium can be restored by decreasing the number of fringe entrants.

constant. The following results show that this is not the case in the bargaining model: as long as the platform is in the hybrid regime, the aggregate decreases in the number of the platform's products.[12]

Theorem 3. *In the bargaining model, if $N_F(N_P) > 0$ (hybrid regime), then the following hold.*

- *The equilibrium number of fringe firms is decreasing in the platform's product variety. Furthermore, this decrease is more than proportional to the increase in the platform's product variety:* $\frac{\mathrm{d}N_F}{\mathrm{d}N_P} < -\frac{V_P}{V_F} < -0$.
- *The equilibrium size of the aggregate and consumer surplus are strictly decreasing in the platform's product variety:* $\frac{\mathrm{d}A}{\mathrm{d}N_P}, \frac{\mathrm{d}CS}{\mathrm{d}N_P} < 0$.

If $N_F(N_P) = 0$ (pure retailer regime), then both the aggregate and consumer surplus are increasing in the platform's product variety:

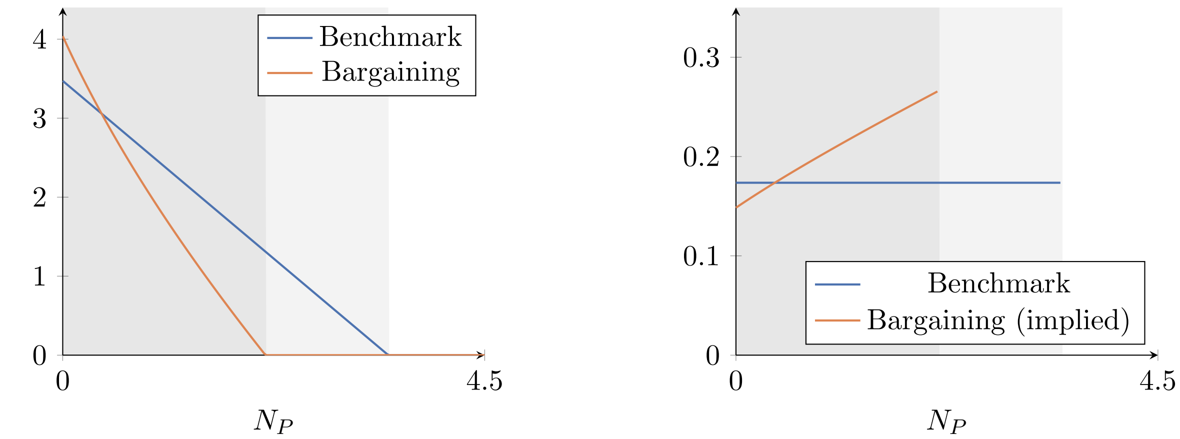
- $\frac{\mathrm{d}A}{\mathrm{d}N_P} = V_P > 0$,
- $\frac{\mathrm{d}CS}{\mathrm{d}N_P} = \frac{\mu V_P}{(1+N_P V_P)^2} > 0$.

The main takeaway from this theorem is that increasing the platform's product variety can decrease consumer welfare in the bargaining model. The underlying reason is illustrated in Figure 4a: the decrease in the number of fringe entrants is large enough to not only to offset the increase in the platform's product variety, but also to decrease the aggregate (Figure 6a). This, in turn, leads to a decrease in consumer surplus (Figure 6b).

This set of results is in stark contrast to the benchmark model. Instead of the increased product variety weakly increasing consumer welfare, when entry fees are negotiated, it has the opposite effect if the platform stays in the hybrid regime.

The intuition behind these results can be understood by examining entry fees. First, define the implied entry fee in the bargaining case as the difference between the variable profit and the

[12] As before, similar results hold for an increase in the platform's product value V_P.



(a) Equilibrium number of fringe firms (N_F)

(b) Optimal/implied entry fee (K_F)

Figure 4. Equilibrium number of fringe entrants and (implied) entry fees ($\mu = 1, V_P = 1, V_F = 1, I_F = 0.05$). In the benchmark model, the entry fee does not depend on the platform's product variety, and an increase in N_P leads to a proportional reduction in N_F. In contrast, in the bargaining case, the entry fee is increasing in N_P, and the reduction in N_F is more than proportional. Dark shaded area represents hybrid mode under the bargaining assumption, while light shaded area represents additional hybrid mode under the benchmark assumption.

final, total profit of the fringe firms (bar the investment cost)[13]:

$$K_F^{impl} = \frac{\pi_F^v - \pi_F^t}{N_F}.$$

As shown before, as the platform's product variety grows, its bargaining power increases. This can be thought of as an increase in (implied) entry fees K_F^{impl}, which in turn discourages fringe entry.

Proposition 9. *In the hybrid regime, the implied entry fee is increasing in the number of the platform's products:*

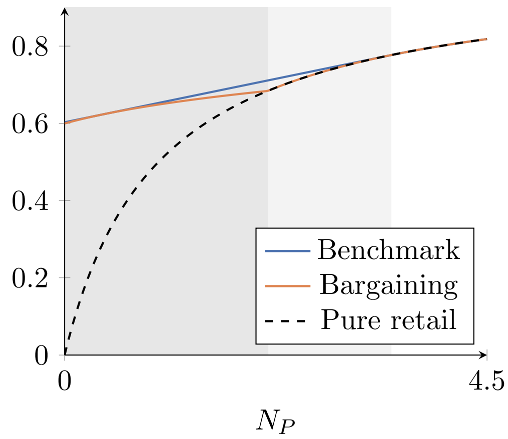
$$N_F(N_P) > 0 \implies \frac{\partial K_F^{impl}(N_P)}{\partial N_P} > 0.$$

This does not happen in the benchmark model because, as stated in Theorem 1, the optimal entry fee does not depend on the platform's product variety. Figure 4b demonstrates this difference between the two models.

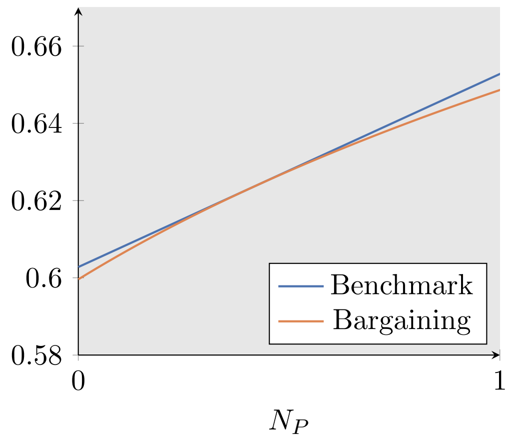
3.4 Platform's mode of operation

While completely endogenizing the platform's mode of operation (i.e., the determination of N_P) is outside the scope of this paper, some insights can be gained by building on the previous results. Let us start by examining whether the platform would prefer to switch to the pure retail regime and exclude the fringe firms.

[13]Or, in light of Appendix C.1, it can also directly be interpreted as the entry fee that the parties agree on after negotiating.

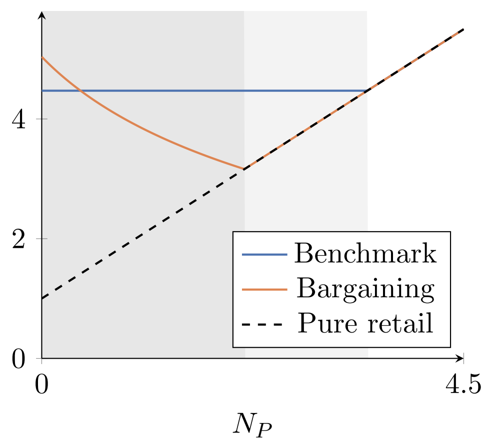


(a)

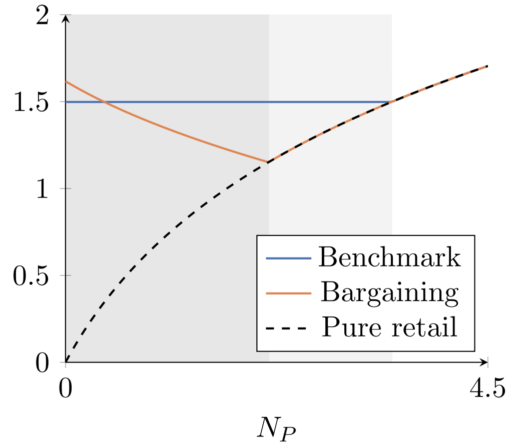


(b)

Figure 5. Platform profits in equilibrium ($\mu = 1, V_P = 1, V_F = 1, I_F = 0.05$). In both cases, platform profits are increasing in N_P. In the case of the hybrid regime under bargaining, this increase is higher than in the benchmark case as long as the implied the entry fee is lower than optimal, and lower afterwards. The right-hand side graph zooms on the relevant section to highlight this observation. Dark shaded area represents hybrid mode under the bargaining assumption, while light shaded area represents additional hybrid mode under the benchmark assumption.



(a) Aggregate (A)



(b) Consumer surplus (CS)

Figure 6. Size of the aggregate and consumer surplus in equilibrium ($\mu = 1, V_P = 1, V_F = 1, I_F = 0.05$). In the benchmark model, an increase in platform product variety has no effect in the hybrid regime. Under bargaining, it does have a negative effect on total product variety, and, in turn, consumer surplus. The increase has a (mechanical) positive effect in the pure retail regime under both assumptions. Dark shaded area represents hybrid mode under the bargaining assumption, while light shaded area represents additional hybrid mode under the benchmark assumption.

Assume that N_P is still fixed, but now, at time 0, the platform can decide whether to exclude the fringe. As the next proposition shows, the platform never has an incentive to do so.[14]

Proposition 10. *In both the benchmark and bargaining models, the platform never has an incentive to switch from the hybrid to the pure retail regime:*

$$\pi_P^t(N_P, N_F(N_P)) \geq \pi_P^t(N_P, 0)\ \forall\, N_P.$$

The inequality is strict whenever $N_F(N_P) > 0$.

This result is straightforward in the case of the benchmark model: if the platform finds it optimal to exclude the fringe, it can always do so by setting the entry fee to a sufficiently high level. In the bargaining case, the reason is less obvious, as it is not immediately clear that the share of profits the platform can negotiate for itself is higher than what it could achieve alone. However, it turns out that this is indeed the case. The intuition is that the platform's profit is the average of total profits, with the average taken over the number of fringe firms. As total profits are increasing in the number of the fringe entrants, this average is also increasing in N_F.

A related, but different question is how the platform would decide between operating in pure retail and pure marketplace modes, if hybrid regime was not an option. It is particularly relevant from a policy perspective, as it can help understand what happens if hybrid operation is banned. The answer to this is less clear-cut, and depends on the parameters of the model. For low enough values of V_P and N_P, the platform chooses the pure marketplace mode, as with pure retail, its profits can be arbitrarily low. The opposite is true for high enough values of V_P and N_P.

Finally, let us look at the more general case, where the platform can choose its number of products at the beginning of the game. Similarly to the fringe, assume that the platform faces an investment cost of I_P per product. Figure 7 illustrates the platform's final profits as a function of its product variety.

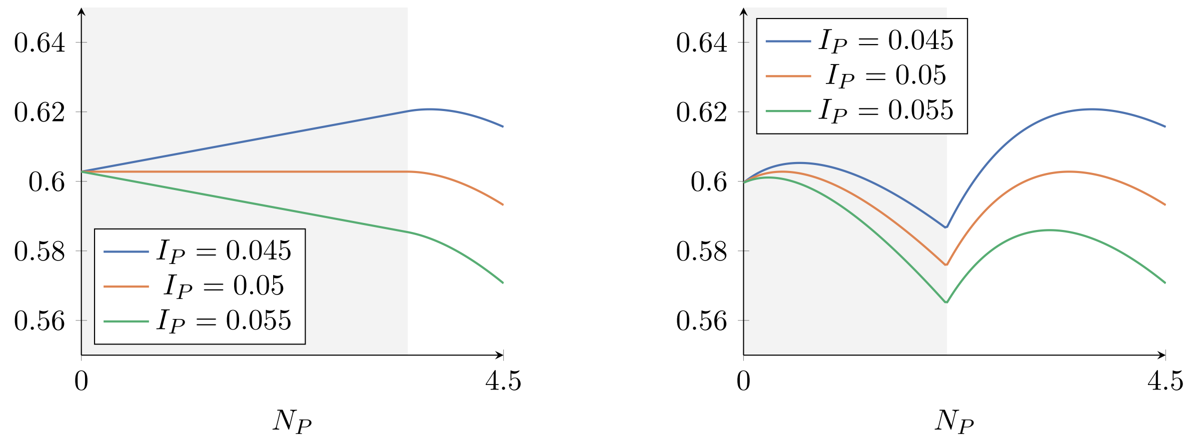
As before, things are straightforward in the benchmark model: the platform invests in its own products only if $\frac{V_P}{I_P} \geq \frac{V_F}{I_F}$. The following proposition characterizes the platform's optimal investment more precisely.

Proposition 11. *Let N_P^* be the platform's optimal number of products. Then, in the benchmark model, the following holds:*

$$\begin{aligned}
\frac{V_P}{I_P} < \frac{V_F}{I_F} &\implies N_P^* = 0, \\
\frac{V_P}{I_P} > \frac{V_F}{I_F} &\implies N_P^* > 0, N_F(N_P^*) = 0, \\
\frac{V_P}{I_P} = \frac{V_F}{I_F} &\implies N_P^* \in [0, \bar{N}_P] \text{ for some } \bar{N}_P > 0.
\end{aligned}$$

The intuition behind this result is that the platform can extract all profits from the fringe firms through the lump-sum entry fee. Thus, it invests in its own products only if it has a

[14] Note that it is still possible that the optimal or negotiated entry fee is such that zero fringe firms enter the market. The proposition's main statement concerns the platform's decision to exclude the fringe when the entry fee would be such that the fringe would enter.



(a) Benchmark case

(b) Bargaining case

Figure 7. Total platform profits if the platform incurs an investment cost for creating its own products. In the benchmark case, hybrid mode operation can only be optimal in the knife-edge case when neither the platform's nor the fringe's product has an advantage. In the bargaining model, hybrid operation may be optimal even if the platform has a product disadvantage. The shaded region represents hybrid operation, while in the unshaded region, the platform operates in pure retail mode ($\mu = 1, V_P = 1, V_F = 1, I_F = 0.05$).

product advantage (either a less costly investment or a more valuable product). A consequence of this proposition is that, apart from the knife-edge case of $\frac{V_P}{I_P} = \frac{V_F}{I_F}$, the platform never finds it optimal to operate in hybrid mode: either it creates so many products that the fringe is not viable, or it creates none at all. Figure 7a displays this result: for low values of I_P, the equilibrium is a corner solution with $N_P = 0$, and for high values, the maximum is reached in the pure retail regime.

As before, the situation is more complicated in the bargaining model. Instead of characterizing the platform's optimal investment directly, let us look at it through the lens of implied entry fees. Remember that in the bargaining model, the implied entry fee increases with the number of the platform's products. Together with the concavity of platform profits as a function of N_P, and the statement of Corollary 1, this implies the following result:

Proposition 12. *In the bargaining model, the dependence of the platform's profits on its number of products is conditional on whether, for a given N_P, the implied entry fee is higher or lower than the optimal entry fee:*

$$K_F^{impl}(N_P) < K_F^{opt} \implies \frac{\mathrm{d}\pi_P^t}{\mathrm{d}N_P} > \frac{V_P}{V_F} I_F$$
$$K_F^{impl}(N_P) > K_F^{opt} \implies \frac{\mathrm{d}\pi_P^t}{\mathrm{d}N_P} < \frac{V_P}{V_F} I_F.$$

The intuition is that if, for some N_P, the implied entry fee is lower than optimal, then the platform investing in more products will bring it closer to the optimal level, and the increase in platform profits will be higher than the increase in the benchmark model. Conversely, if the entry fee is already higher than optimal for a given N_P, an additional increase will lead to an even more suboptimal (implied) entry fee and, thus, a lower increase in platform profits.

This result implies that in the bargaining model, hybrid operations can be optimal, and not only for knife-edge cases. Furthermore, and even more strikingly, the platform may find it optimal to invest in its own products even if it has a product disadvantage, when this investment helps it to negotiate a more optimal entry fee. Figure 7b showcases such an example. When the platform has a not-too-large product disadvantage ($I_P = 0.55$ case), its profits are maximized by operating in the hybrid regime. This result is doubly unfortunate from a welfare perspective: not only does the platform make an investment that the fringe could have made more efficiently, but it also decreases consumer welfare by reducing the total product variety.

3.5 Discussion

Generality of the figures While Figures 4 to 6 illustrate the main results of this paper under a specific parametrization, many of the insights are more general. In particular, the higher-than-proportional decrease in the number of fringe firms and the resulting negative effect on consumer welfare have corresponding theorems that hold for any parametrization (Theorems 2 and 3). Similarly, the implied entry fee increasing in the number of the platform's products is also a general result (Proposition 9).

One aspect that is not general is the fact that the optimal and implied entry fees, in the benchmark and bargaining models, respectively, coincide for some value of N_P. In the example parametrization, the implied entry fee is lower than what the platform would prefer to set for low values of N_P and higher afterward. Therefore, there is an optimal platform product variety N_P^* for which the implied entry fee is the same as the optimal entry fee in the benchmark model, and the results of the two models are the same. However, it is possible that even for arbitrarily low values of N_P, the implied entry fee is higher than optimal. In such a case, the platform would like to commit to a lower entry fee to incentivize more fringe entry, but it cannot do so. Appendix A.1 presents an example where this is the case. In such a setting, bargaining does not improve consumer welfare compared to unilateral price-setting, even if the platform is a pure marketplace: it can be a lose-lose situation for both the platform and the consumers.

Consequences of the assumptions The results from the benchmark model suggest that the platform having its own products is always weakly beneficial for consumers. Furthermore, if the platform's product variety (and thus also the pure marketplace/ hybrid mode decision) was endogenous, an improvement in the platform's product (either higher value or lower cost) would also have a positive effect on consumer welfare. This surprising result depends on a number of – admittedly unrealistic – assumptions. Namely, the lump-sum nature of the entry fee, no entry fee for consumers, and the platform pricing its products as if separate sellers produced them. Therefore, this result should not be taken as a conclusion applicable to the real world but as a best-case benchmark. On the other hand, these assumptions are also why this model is so useful as a benchmark: as I show in the case of the bargaining model, even with these assumptions in place, the platform selling its own products can have negative welfare consequences when the entry fee is set through bargaining.

Nonetheless, the results of the bargaining model do not hinge on the majority of these assumptions. For example, the number of consumers could be endogenized, with the platform setting an entry fee for them. Similarly, the platform could take into account the fact that its product prices affect the demand for its other products. These changes would translate into a different, possibly less tractable total profit function, but its main properties (notably, being increasing in the number of platform products and fringe firms) would still be the same. As long as Lemma 1 holds, the main results of the bargaining model would also be valid.

Furthermore, the logit-like nature of the demand function is also not necessary. It does simplify the analysis, especially in terms of consumer welfare, but the main results would also hold for a larger class of demand functions[15]. It is, however, important that the demand function is such that total profits are increasing in the number of fringe firms. Without this, the corresponding cooperative game would not be monotone, and the bargaining interpretation would be less applicable. In the model presented in the main text, monotonicity arises due to consumers' taste for variety preferences. This assumption might also be justified in settings with network effects (Rochet and Tirole 2003), or when the platform can extract consumer surplus through entry fees or some other mechanism.[16]

The other assumption crucial for the results is the lump-sum nature of the entry fee. Without it, the corresponding cooperative game would not have transferable utility, and the Shapley value would not be a meaningful solution concept. One alternative solution concept for that case would be the consistent Shapley value from Maschler and Owen (1992), which can also be interpreted as a bargaining solution (S. Hart and Mas-Colell 1996).

Relation to the literature Similar to the existing literature on hybrid platforms, this paper highlights the problematic aspects of hybrid operation, albeit through a novel channel. In Hagiu, Teh, and Wright (2022), the negative consequences are due to its incentives to engage in anti-competitive behavior, such as self-preferencing or imitation. However, in lieu of those, they argue that hybrid platforms can be beneficial. Anderson and Bedre-Defolie (2021), on the other hand, shows that even in the absence of such behavior, hybrid platforms can harm consumers, as platforms are inclined to set higher than optimal royalties to create a price advantage for their own products. The distortion in their model comes from the percentage-based nature of the entry fee (royalty), which incentivizes the platform to set it higher than in the pure intermediary case to give an advantage to its own products. This paper goes one step further. Due to the lump-sum nature of the entry fee and platforms pricing their products as if they were produced by separate sellers, the latter incentive is no longer present (as demonstrated by the benchmark model). However, I demonstrate that there is another source of distortion when the entry fee is negotiated: a hybrid platform is in a better bargaining position than it would be if it had no products on its own. This, in turn, leads to a higher entry fee and, thus, lower consumer welfare.

4 Conclusion and future work

This paper introduces a new model of hybrid platforms in which bargaining between the platform and the entrant firms plays a key role. It highlights the importance of a so-far overlooked

[15] In particular, demand functions for which the implied profit functions satisfy the assumptions in Appendix B.

[16] Chen and Riordan (2008) demonstrates that under certain circumstances, an even more extreme phenomenon might occur: competition may increase not only industry-wide profits but prices, as well.

aspect of platforms having their own products: the fact that it increases their bargaining power compared to other players. In certain situations, such as the one described in this paper, this can lead to fewer fringe reduced product variety and, ultimately, lower consumer welfare. As a result, even in the absence of other frictions, hybrid platforms might have detrimental effects on fringe entry.

The demand structure is similar to the one in Anderson and Bedre-Defolie (2021), and the results convey the same general message: hybrid platforms can have detrimental welfare effects. However, the mechanism behind this result is quite different. This paper showcases the channel of changes in bargaining power and shows that even in the case of lump-sum entry fees, hybrid platforms can be problematic from a welfare perspective. An extension of the main model also highlights the importance of the assumptions on who is participating in the bargaining process.

As shown in the appendix, the model can be generalized beyond the hybrid platform setting, and can describe other markets, such as upstream-downstream relations or franchising, as well. The generality of the results obtained also highlights the similarities between such markets. Furthermore, many of those results from can be applied in a plug-and-play fashion to other models of large-player-small-player bargaining.

There are several avenues for future research in this direction. First, although the results provide some suggestions about what endogenizing the number of the platform's products might entail, a more formal analysis is needed for a more complete picture. Second, applying the same bargaining framework but using a different demand structure (e.g., CES utility) for microfounding the profit functions would have important implications for the robustness of the results. Finally, I believe this approach for modeling bargaining is a rather good compromise between assuming that one party has all the bargaining power and modeling the bargaining process in detail. Therefore, certain ideas in this paper might be a good fit for modeling different settings involving bargaining power disparities.

References

Aghion, Philippe, Rachel Griffith, and Peter Howitt (2006). "Vertical integration and competition". In: *American Economic Review* 96.2, pp. 97–102.

Anderson, Simon P and Özlem Bedre-Defolie (2021). "Hybrid platform model". In.

Anderson, Simon P, Nisvan Erkal, and Daniel Piccinin (2020). "Aggregative games and oligopoly theory: short-run and long-run analysis". In: *The RAND Journal of Economics* 51.2, pp. 470–495.

Armstrong, Mark (2006). "Competition in two-sided markets". In: *The RAND journal of economics* 37.3, pp. 668–691.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky (1986). "The Nash bargaining solution in economic modelling". In: *The RAND Journal of Economics*, pp. 176–188.

Brügemann, Björn, Pieter Gautier, and Guido Menzio (2019). "Intra firm bargaining and Shapley values". In: *The Review of Economic Studies* 86.2, pp. 564–592.

Calvano, Emilio and Michele Polo (2021). "Market power, competition and innovation in digital markets: A survey". In: *Information Economics and Policy* 54, p. 100853.

Chen, Yongmin and Michael H Riordan (2008). “Price-increasing competition”. In: *The RAND Journal of Economics* 39.4, pp. 1042–1058.

Council of European Union (2022). *Regulation (EU) 2022/1925 of the European Parliament and of the Council of 14 September 2022 on contestable and fair markets in the digital sector and amending Directives (EU) 2019/1937 and (EU) 2020/1828 (Digital Markets Act)*. URL: http://eur-lex.europa.eu/legal-content/EN/TXT/?qid=1416170084502&uri=CELEX:32014R0269.

Crawford, Gregory S, Robin S Lee, Michael D Whinston, and Ali Yurukoglu (2018). “The welfare effects of vertical integration in multichannel television markets”. In: *Econometrica* 86.3, pp. 891–954.

De Fontenay, Catherine C and Joshua S Gans (2005). “Vertical integration in the presence of upstream competition”. In: *RAND Journal of Economics*, pp. 544–572.

Dou, Yifan (2014). “When to sell premium digital contents through the rival platform?” In: *2014 11th International Conference on Service Systems and Service Management (ICSSSM)*. IEEE, pp. 1–6.

Evans, David S, Richard Schmalensee, Michael D Noel, Howard H Chang, and Daniel D Garcia-Swartz (2011). “Platform economics: Essays on multi-sided businesses”. In: *Platform Economics: Essays on Multi-Sided Businesses, David S. Evans, Ed., Competition Policy International*.

Fletcher, Amelia, Gregory S Crawford, Jacques Crémer, David Dinielli, Paul Heidhues, Michael Luca, Tobias Salz, Monika Schnitzer, Fiona M Scott Morton, Katja Seim, et al. (2021). “Consumer Protection for Online Markets and Large Digital Platforms”. In.

Gul, Faruk (1989). “Bargaining foundations of Shapley value”. In: *Econometrica*, pp. 81–95.

Gutierrez, German (2021). “The welfare consequences of regulating Amazon”. In: *Job Market Paper, New York University*.

Hagiu, Andrei (2004). “Optimal pricing and commitment in two-sided markets”. In: *Rand Journal of Economics* 20, pp. 658–670.

Hagiu, Andrei and Robin S Lee (2011). “Exclusivity and control”. In: *Journal of Economics & Management Strategy* 20.3, pp. 679–708.

Hagiu, Andrei, Tat-How Teh, and Julian Wright (2022). “Should platforms be allowed to sell on their own marketplaces?” In: *The RAND Journal of Economics* 53.2, pp. 297–327.

Hart, Oliver and John Moore (1990). “Property Rights and the Nature of the Firm”. In: *Journal of political economy* 98.6, pp. 1119–1158.

Hart, Oliver, Jean Tirole, Dennis W Carlton, and Oliver E Williamson (1990). “Vertical integration and market foreclosure”. In: *Brookings papers on economic activity. Microeconomics* 1990, pp. 205–286.

Hart, Sergiu and Andreu Mas-Colell (1996). “Bargaining and value”. In: *Econometrica*, pp. 357–380.

Ho, Kate and Robin S Lee (2017). “Insurer competition in health care markets”. In: *Econometrica* 85.2, pp. 379–417.

Inderst, Roman and Christian Wey (2003). “Bargaining, mergers, and technology choice in bilaterally oligopolistic industries”. In: *RAND Journal of Economics*, pp. 1–19.

Kang, Zi Yang and Ellen V Muir (2022). “Contracting and vertical control by a dominant platform”. In: *Unpublished manuscript, Stanford University*.

Lee, Robin S (2013). “Vertical integration and exclusivity in platform and two-sided markets”. In: *American Economic Review* 103.7, pp. 2960–3000.

— (2014). “Competing platforms”. In: *Journal of Economics & Management Strategy* 23.3, pp. 507–526.

Levy, Anat and Lloyd S Shapley (1997). “Individual and collective wage bargaining”. In: *International Economic Review*, pp. 969–991.

Livni, Ephrat and Michael J. De La Merced (Feb. 2023). *Microsoft’s Activision deal tests a new global alignment on antitrust*. URL: https://www.nytimes.com/2023/02/04/business/dealbook/microsofts-activision-deal.html.

Maschler, Michael and Guillermo Owen (1992). “The consistent Shapley value for games without side payments”. In: *rational interaction: Essays in Honor of John c. Harsanyi*, pp. 5–12.

Montez, João V (2007). “Downstream mergers and producer’s capacity choice: why bake a larger pie when getting a smaller slice?” In: *The RAND Journal of Economics* 38.4, pp. 948–966.

Phartiyal, Sankalp (Jan. 2019). *Explainer: What are India’s new foreign direct investment rules for e-commerce?* URL: https://www.reuters.com/article/us-india-ecommerce-explainer-idUSKCN1PP1Y2.

Reynolds, Stefani (Mar. 2022). *WSJ News Exclusive — antitrust bill targeting Amazon, Google, Apple gets support from DOJ.* URL: https://www.wsj.com/articles/doj-backs-antitrust-bill-targeting-amazon-google-apple-11648519385?mod=tech_lead_pos1.

Rochet, Jean-Charles and Jean Tirole (2003). “Platform competition in two-sided markets”. In: *Journal of the European Economic Association* 1.4, pp. 990–1029.

— (2006). “Two-sided markets: a progress report”. In: *The RAND journal of economics* 37.3, pp. 645–667.

Shapley, Lloyd S (1953). *Additive and non-additive set functions*. Princeton University.

Small, Kenneth A and Harvey S Rosen (1981). “Applied welfare economics with discrete choice models”. In: *Econometrica: Journal of the Econometric Society*, pp. 105–130.

Stancsics, Martin (2024). “The value of being indispensable: a cooperative approach to bargaining with a continuum of players”.

Steiner, Robert L (2004). “The nature and benefits of national brand/private label competition”. In: *Review of Industrial Organization* 24.2, pp. 105–127.

Stole, Lars A and Jeffrey Zwiebel (1996a). “Intra-firm bargaining under non-binding contracts”. In: *The Review of Economic Studies* 63.3, pp. 375–410.

— (1996b). “Organizational design and technology choice under intrafirm bargaining”. In: *The American Economic Review*, pp. 195–222.

Weber, Robert J (1988). “Probabilistic values for games”. In: *The Shapley Value. Essays in Honor of Lloyd S. Shapley*, pp. 101–119.

Weeds, Helen (2016). “TV wars: Exclusive content and platform competition in pay TV”. In: *The Economic Journal* 126.594, pp. 1600–1633.

Winter, Eyal (1994). “The demand commitment bargaining and snowballing cooperation”. In: *Economic Theory* 4.2, pp. 255–273.

A Extensions

A.1 Players have different innate bargaining power

In the previous sections, I assumed that the platform and the fringe firms split total profits according to their Shapley values. The symmetry property of the Shapley value excludes the possibility of players having some innate bargaining power which is not related to their profit functions. In this extension, I will relax this assumption and assume instead that profits are shared according to *weighed* values. These are a generalization of the Shapley value, where each player has a weight w_i that can, in certain settings, be considered a parameter describing bargaining power.[17] I still assume that fringe firms are identical, also in regard to their bargaining weights. Therefore, the only difference is between the platform and the fringe firms. Let us denote the platform's bargaining weight as $\lambda_P \in \mathbb{R}^+$, and without loss of generality, normalize the fringe firms' weights to 1.

As shown in Appendix C.2.1, the weighted Shapley value, and thus final profits in this setting, is given by

Proposition A.1. *The Shapley value of the platform and the fringe firms, and thus final total profits, are given by:*

$$\pi_P^t(N_P, N_F) = \int_0^1 \lambda_P s^{\lambda_P - 1} \Pi(N_P, sN_F) \mathrm{d}s,$$
$$\pi_F^t(N_P, N_F) = \int_0^1 s^{\lambda_P} \partial_2 N_F \Pi(N_P, sN_F) \mathrm{d}s.$$

Proof of Proposition A.1. □

This result is a direct consequence of Proposition C.2. It is similar to the non-weighted value in that it is an average of marginal contributions.However, now those averages are weighted, with the weight function depending on the platform's bargaining weight.

The interpretation of λ_P as a bargaining weight for the platform is supported by the fact that the platform's profit function is increasing in it.

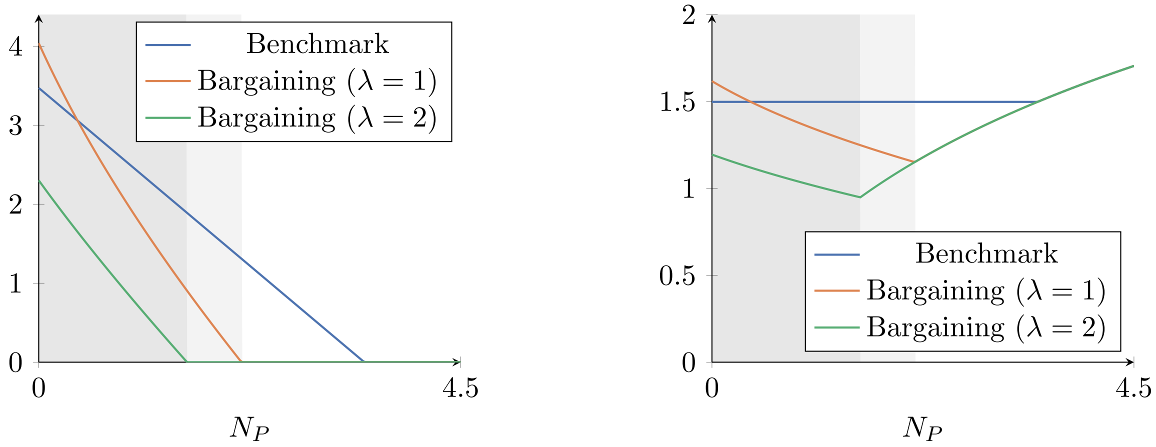
Proposition A.2. *For any fixed $N_F > 0, N_P \geq 0$, the platform's total profits are increasing in its bargaining weight λ_P, while the fringe's profits are decreasing in it:*

$$\frac{\partial \pi_P^t(N_P, N_F)}{\partial \lambda_P} > 0,$$
$$\frac{\partial \pi_F^t(N_P, N_F)}{\partial \lambda_P} < 0.$$

Proof of Proposition A.2. For any $\lambda < \lambda'$, $\int_0^s \lambda t^{\lambda-1} \mathrm{d}t = s^\lambda > s^{\lambda'} = \int_0^s \lambda' t^{\lambda'-1} \mathrm{d}t \forall s \in [0,1]$. Therefore, for any (non-constant) function Π, $\int_0^1 \lambda s^{\lambda-1} \Pi(s) \mathrm{d}s > \int_0^1 \lambda' s^{\lambda'-1} \Pi(s) \mathrm{d}s$. □

In fact, the limits as $\lambda_P \to 0$ and $\lambda_P \to \infty$ are quite intuitive: in the former case, the platform's profits are zero, while in the latter case, it can appropriate all of the profits. That is, these limits correspond to the platform either receiving or making take-it or leave-it offers.

[17] S. Hart and Mas-Colell (1996) and Stole and Zwiebel (1996a) provide foundations for this interpretation.



(a) Equilibrium number of fringe firms (N_F)

(b) Equilibrium consumer surplus (CS)

Figure A.1. Equilibrium outcomes in the case when the platform has higher innate bargaining power ($V_P = 1, V_F = 1, I_F = 0.05$). As before, consumer surplus is decreasing in N_P as a result in a decrease in total product variety. Dark shaded area represents hybrid mode under $\lambda = 2$, while light shaded area represents additional hybrid mode under $\lambda = 1$.

The example parametrization is the same as in the main model, with the only difference being that the platform has a higher innate bargaining power ($\lambda_P = 2$). The main result, namely that increasing the platform's product variety has a negative effect on consumer welfare in the hybrid regime, still holds. That is, as shown on Figure A.1a, the total aggregate is decreasing in N_P throughout the hybrid regime due to the platform's products displacing more fringe products than their total number.

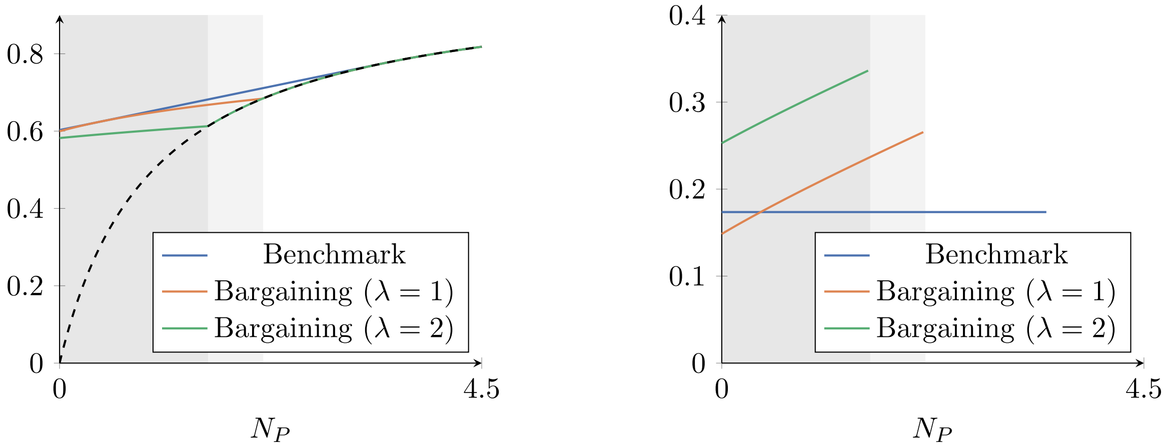
The main difference is that, for any $N_P \geq 0$, the implied entry fee (Figure A.2a) is higher than the benchmark, unilaterally set one, due to the higher bargaining power of the platform. That is, for any N_P, the platform would prefer to set a lower entry fee, but it is unable to do so due to its high bargaining power. Total aggregate, and thus consumer surplus, is also below the main model's outcome in this case.

Another significant difference pertains to the total profits of the platform as a function of N_P. It is still true that they are increasing in the number of the platform's products, but now this increase is slower in the hybrid regime. The reason is that the implied entry fee is always higher than the optimal one, therefore the positive effect of an increase in N_P is somewhat counterbalanced by the entry fee becoming even more suboptimal.

A.2 Three sided bargaining

In the second extension, I consider the case when the other side also participates in the bargaining process. This is more plausible in business-to-business settings, so I use the term customers instead of consumers in this section. Nevertheless, I use the same demand structure as in the main model.

This extension entails two changes compared to the main model. First, the players bargain over the total surplus generated on all sides of the market, not just the total profits from selling the products. Second, the outcomes are assumed to be described by a cooperative game with



(a) Platform profits

(b) Optimal/implied entry fee

Figure A.2. Platform profits and (implied) entry fees ($\mu = 0.2, V_P = 1, V_F = 1, I_F = 0.05$). Regardless of lambda, platform profits and entry fees are increasing in N_P. However, when the platform's innate bargaining power is higher, entry fees are above optimal levels already for $N_P = 0$, and the additional increase somewhat mitigates the benefits of the platform's increased product variety. When λ is lower, the entry fees are below optimal for low N_P, therefore increasing N_P has a larger positive effect on profits. Dark shaded area represents hybrid mode under $\lambda = 2$, while light shaded area represents additional hybrid mode under $\lambda = 1$.

three types of players: the platform, the fringe firms, and the consumers.

Let us start by deriving total surplus as a function of N_P, N_F, and N_C (the number of customers). Under the logit-like demand structure described in Section 2.2.1, the total surplus is given by

Proposition A.3. *Assuming profit-maximizing prices from the platform and the fringe, total surplus as a function of N_P, N_F and N_C is given by*

$$\Pi(N_P, N_F, N_C) = \mu N_C \left[\frac{N_P V_P + N_F V_F}{N_P V_P + N_F V_F + 1} + \log(N_P V_P + N_F V_F + 1) \right].$$

Proof of Proposition A.3. See Small and Rosen (1981) for a derivation for the discrete case. The continuous case is analogous. □

Note that this expression is of the form $\Pi(N_P, N_P, N_C) = N_C f(N_P, N_F)$. Furthermore, $f(N_P, N_F) = g(V_P N_P + V_F N_F)$ for an increasing, strictly concave g.

Next, let us consider the bargaining outcomes in this three-sided setting. Appendix C.2.2 formally describes the corresponding cooperative game, and Proposition C.3 establishes the resulting profit shares. To summarize, the various players share the total surplus in the following way.

Proposition A.4.

$$\pi_P(N_P, N_F, N_C) = \int_0^1 s\Pi(N_P, sN_F)\mathrm{d}s,$$
$$\pi_F(N_P, N_F, N_C) = \int_0^1 s^2 N_F \partial_2\Pi(N_P, sN_F)\mathrm{d}s$$
$$CS(N_P, N_F, N_C) = \int_0^1 s\Pi(N_P, sN_F)\mathrm{d}s.$$

Proof of Proposition A.4. Appendix C.2.2 formally describes the cooperative game corresponding to the three-sided bargaining assumption. This result is a direct consequence of Proposition C.3. □

There are a number of things to note here. First, and most importantly, the customers' share from total surplus is equal to the platform's.[18] As a consequence, what is good for the platform is also beneficial for the customers. Therefore, if the platform can choose its mode of operation to maximize its profits, it also maximizes the customers' surplus (but not necessarily total surplus).

Second, it can be shown that due to the fringe firms' total profit function having a similar shape as in the main model, the same results hold in terms of the platform's product variety displacing fringe products.

Proposition A.5. *In the hybrid regime under three-sided bargaining, the following holds:*

$$N_F > 0 \implies \frac{\mathrm{d}N_F}{\mathrm{d}N_P} < -\frac{V_P}{V_F}.$$

Proof of Proposition A.5. The proof is analogous to that of Theorem 3. To see that $\frac{\partial \pi_F}{\partial N_F}$ satisfies Assumption B.5, first consider the function

$$g(x) = \frac{x}{1+x} + \log(1+x).$$

Then observe that

$$G(x) :- \int_0^1 g(sx)\mathrm{d}s = \frac{\frac{x^2(x+1)}{2} + (1 - \log(x+1))(x+1) - 1}{x^2(x+1)}$$

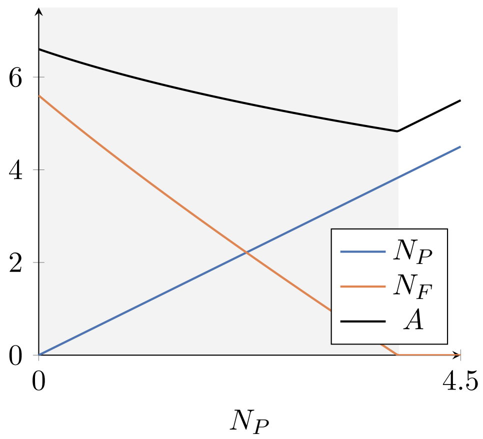
is concave, and thus satisfies Assumption B.5. By Lemma B.1, π_F then also satisfies Assumption B.5. Finally, as Assumptions B.1 to B.3, B.5 and B.6 are satisfied, it follows from Proposition B.5 that

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} < -\frac{V_P}{V_F}$$

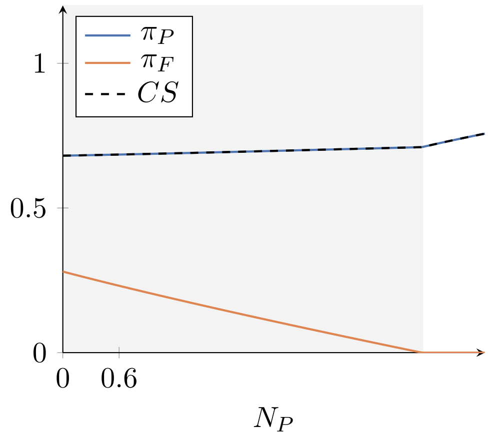
in the hybrid regime.

□

[18] This is a consequence of the fact that customers are ex ante identical, and each of them buy one product, therefore demand, and in turn total surplus is linear in N_C.



(a) Platform product variety and number of fringe entrants



(b) Platform and fringe profits

Figure A.3. Equilibrium outcomes in the case when the whole surplus is bargained over ($N_C = 0.6, V_P = 1, V_F = 1, I_F = 0.05$), and both consumers and fringe firms participate in the bargaining. As before, the platform's profits are increasing in N_P, while the fringe's profits are decreasing. On the other hand, consumer welfare is equal to platform profits, therefore it is increasing in N_P. The shaded region represents hybrid operation, while in the unshaded region, the platform operates in pure retail mode.

As a consequence, the aggregate is decreasing in N_P in the hybrid regime (Figure A.3a).

Corollary A.1. *In the hybrid regime under three-sided bargaining, total aggregate is decreasing in the platform's product variety:*

$$N_F > 0 \implies \frac{\partial A}{\partial N_P} < 0.$$

Proof. This result also follows from the applicability of Proposition B.5. □

Nevertheless, there is an important distinction compared to the previous results. While this does decrease total surplus, in contrast to the two-sided bargaining case, it does not imply a decrease in the customers' surplus. Their share of the total surplus is equal to the platform's profits, and thus, it is increasing in N_P if and only if platform profits (Figure A.3b) are. Therefore, customers might prefer hybrid operation to the platform being a pure marketplace.

B Generalization and proofs

While the main text examines bargaining between the platform and the entrants in the context of a specific, logit-like demand system, many of the results are more general. This section presents those and the necessary assumptions. As the results in the main text are not proven directly, but rather derived from the general results, the proofs are also included here.

B.1 Production

Let us assume the following reduced-form, but rather general total profit (or, where applicable, surplus) function:

Assumption B.1. *The total profits of the platform and the fringe are described by*

$$\Pi(N_P, N_F, N_C) = N_C f(N_P, N_F),$$

where $f : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$.

It can be justified for markets with the following features: (1) all fringe firms are identical, so only their number matters in terms of total profit, and (2) consumers are also identical (bar their idiosyncratic taste shocks).

I will assume that, in addition to being increasing in the number of consumers, total profits are also increasing in both the number of fringe firms and the platform' products.

Assumption B.2. $f(N_P, n_F)$ *is increasing in both* N_P *and* N_F.

Such profit functions arise in settings where the profit reduction from increased competition is dominated by extra sales due to increased product variety. Section 3.5 discusses this assumption in more detail.

B.2 Profit sharing

Next, assume that the platform and the fringe share profits according to the following rule:

Assumption B.3. *Let* $w_P, w_F : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ *be non-negative functions such that the following condition holds:* $\pi_P(N_P, N_F, N_C) + \pi_F(N_P, N_F, N_C) \leq \Pi(N_P, N_F, N_C)$. *Then, the platform's profit* ($\pi_P(N_P, N_F, N_C)$) *and the fringe's profit* ($\pi_F(N_P, N_F, N_C)$) *are given by:*

$$\pi_P(N_P, N_F, N_C) = N_C \int_0^1 w_P(s) f(N_P, sN_F) \mathrm{d}s,$$
$$\pi_F(N_P, N_F, N_C) = N_C \int_0^1 w_F(s) N_F \partial_2 f(N_P, sN_F) \mathrm{d}s.$$

It covers the cases in the main text and the extensions (namely, the Shapley value, the weighted value, and three-way bargaining) but is also more general than those. In particular, in the platform game, such a rule can describe any random order value (Weber 1988).

In the main text, I use a simpler version of this profit sharing rule. I assume that $w_P(s) = w_F(s) \equiv 1$. As Appendix C.2 demonstrates, this corresponds to players getting their Shapley values. Together with the assumptions on the demand system, one can even derive a closed-form expression for the platform's profit share (Proposition 5). The proof of this proposition is given below.

Proof of Proposition 5. Simply integrate the total industry profit function with respect to the mass of fringe entrants obtain the platform's share of the pie:

$$\begin{aligned}\pi_P^t &= \int_0^1 \Pi(N_P, sN_F)\mathrm{d}s \\ &= \mu \int_0^1 \frac{N_P V_P + sN_F V_F}{N_P V_P + sN_F V_F + 1}\mathrm{d}s \\ &= \mu \left[1 - \frac{\log\left(1 + \frac{N_F V_F}{N_P V_P + 1}\right)}{N_F V_F} \right].\end{aligned}$$

The fringe's share is just the remainder,

$$\Pi(N_P, N_F) - \int_0^1 \Pi(N_P, sN_F)\mathrm{d}s = \mu \left[\frac{\log\left(1 + \frac{N_F V_F}{N_P V_P + 1}\right)}{N_F V_F} - \frac{1}{N_P V_P + N_F V_F + 1} \right]$$

Finally, remember that the investment cost of the fringe is fixed at the bargaining stage, and is therefore not included in the bargaining outcome. Therefore, the total profits of the complete fringe are

$$\pi_P^t = \mu \left[\frac{\log\left(1 + \frac{N_F V_F}{N_P V_P + 1}\right)}{N_F V_F} - \frac{1}{N_P V_P + N_F V_F + 1} \right] - I_F N_F.$$

□

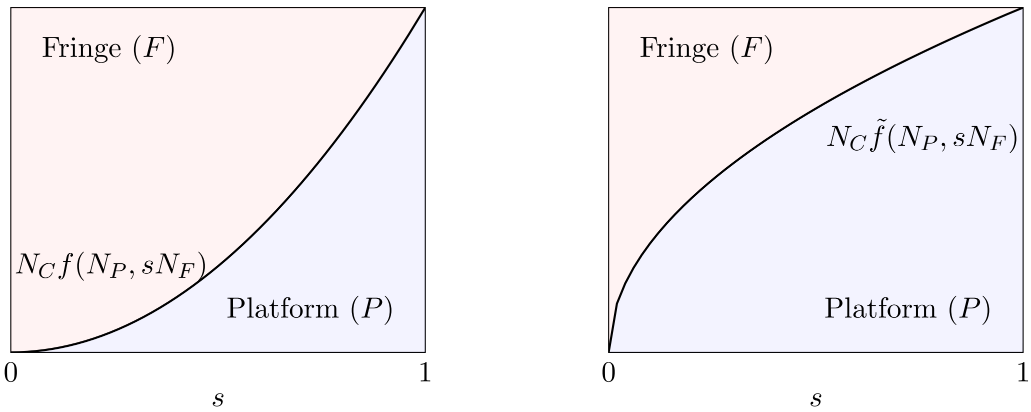
Apart from the cooperative foundations, one primary justification for this profit allocation rule is its intuitive behavior in terms of comparative statics. To illustrate this, let us examine what happens when one varies the substitutability between the fringe firms. As it turns out, the platform's share increases when the fringe firms are more substitutable. The following observation demonstrates this idea.

Proposition B.1. *Assume that Assumptions B.1 to B.3 hold. Fix some $N_P, N_C \geq 0$. Let $f, \tilde{f} : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ two different profit functions such that $f(N_P, N_F) = \tilde{f}(N_P, N_F)$ for some N_P, N_F and $f(N_P, n_F) \leq \tilde{f}(N_P, n_F)$ for all $n_F < N_F$. Furthermore, let us denote the corresponding platform profit shares by π_P and $\tilde{\pi}_P$.*

Then, $\pi_P \leq \tilde{\pi}_P$.

Proof of Proposition B.1. Immediately follows from the monotonicity of the integral. □

In words, Proposition B.1 describes two alternative worlds in which N_F fringe firms and a platform with N_P product variety can achieve the same total profit level. However, in the case with $\tilde{f}$, the fringe firms are more substitutable to each other in the sense that fewer of them are needed to achieve a given level of profit (see Figure B.1). The observation is that, in this situation, the platform's share is indeed higher when the fringe firms are more substitutable. This coincides with the intuitive idea that the platform's bargaining power is higher when it does not mind losing a few fringe sellers.



(a) Fringe firms are complements

(b) Fringe firms are substitutes

Figure B.1. Distribution of value between the platform and the fringe. Profit shares correspond to the shaded areas. The platform's profit share is higher when the fringe firms are more substitutable.

B.3 Equilibrium

Let us now turn to determining the equilibrium number of fringe firms. Assume that each firm faces a lump-sum investment cost of I_F to enter the market.[19] Thus, the total investment cost for the fringe is given by $I_F N_F$. The equilibrium number of entrants is determined by a free entry condition: in the end, the fringe firms' total profits should be equal to the aggregated investment cost.

Assumption B.4. *Let us define a free entry equilibrium by the following conditions: Entrants make zero profits after accounting for entry costs:*

$$\pi_F(N_P, N_F) = I_F N_F.$$

Assumption B.5. *Finally, in order to guarantee a unique equilibrium, let us make the following additional assumption about the profit function. Let f be such that the following holds for all $N_F, N_P, N_C \geq 0$:*

$$\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F} < 0 \text{ or } \frac{\partial^2 \pi_F(N_P, N_F, N_C)}{\partial N_F^2} < 0$$

This assumption essentially guarantees that the profit of the fringe (as a function of the number of entrants) has at most one single crossing with total entry cost (apart from the obvious $N_F = 0$ intersection). This assumption is satisfied under the specific demand system and profit-sharing rule described in the main text.

Proof of Lemma 1. First, consider the function

$$g(x) = \frac{sx}{1 + sx}.$$

[19] The results would also hold if the investment costs were non-constant, as long as the marginal cost of investment is weakly increasing.

Also, let us define

$$\begin{aligned} G(x) &= \int_0^1 sg(sx)\mathrm{d}s \\ &= \frac{\log(x+1)}{x} + \frac{1}{x(x+1)} - \frac{1}{x}. \end{aligned}$$

Differentiation with respect to x yields that

$$G'(x) < 0 \iff \log(x+1) > \frac{x(2x+1)}{(x+1)^2}$$

and

$$G''(x) < 0 \iff \log(x+1) < \frac{x(5x^2+5x+2)}{2(x+1)^3}.$$

There exists $\bar{x} > 0$[20], such that the first equality holds for all $x > \bar{x}$, and the second for all $x < \bar{x}$. Therefore, G is either concave or decreasing for all $x > 0$.

Now let us consider the function $h(x) = N_C \mu g(N_P V_P + V_F x)$. By Lemma B.1, $H(x) = \int_0^1 sh'(sx)\mathrm{d}s$ is also either concave or decreasing for any $x > 0$. Finally, notice that $H(x)$ is exactly the profit function of the fringe:

$$\pi_F(N_P, N_F) = \int_0^1 N_F h'(sN_F)\mathrm{d}s,$$

thus proving the lemma. □

With Assumption B.5 in place, the uniqueness of the equilibrium can be established.

Proposition B.2. *Under the conditions in Assumptions B.4 and B.5, the equilibrium is unique if it exists.*

Proof of Proposition B.2. Lemma B.2 states that for any positive N_F^* for which $\pi_F(N_P, N_F^*) - N_F I_F$, the partial derivative with respect to the number of fringe firms is negative. Now assume by contradiction that $\exists 0 < N_F^* < N_F^{**}$ such that $\pi_F(N_P, N_F^*) = I_F N_F^*$ and $\pi_F(N_P, N_F^{**}) = I_F N_F^{**}$. But then the mean value theorem implies that there is a $\bar{N}_F \in (N_F^*, N_F^{**})$ such that $\partial_F \pi_F(N_P, \bar{N}_F) = I_F$. This is a contradiction, as $\partial_F \pi_F(N_P, N_F) < I_F$ for all $N_F^* < N_F$. □

The intuition behind this result is that Assumption B.5 ensures that the total profits achieved by the fringe are either concave or hump-shaped. Consequently, it has at most one crossing with the – convex and increasing – total entry cost function (for $n_F > 0$). This particular shape (concave or hump-shaped) is also the main driver for the later comparative statics results of equilibrium profits and number of entrants. Given that Assumptions B.4 and B.5 is satisfied in the model presented in the main text, it immediately follows that the equilibrium in that model is also unique.

[20] $\bar{x} = 3$ is such a number.

Proof of Proposition 7. Lemma 1 demonstrates that the fringe total profit function is either concave or decreasing in the number of fringe firms. Thus, Assumption B.5 is satisfied, and Proposition B.2 implies that the equilibrium number of fringe entrants is unique. □

B.4 Comparative statics

This section presents a number of comparative statics results that can be obtained even in this rather abstract setting. It contains three sets of results: (1) participants' profits in a partial equilibrium setting, where the number of fringe firms is taken as fixed, (2) equilibrium entry as a function of the platform's product variety, and (3) the platform's profits in general equilibrium. Throughout the paper, I use $\frac{\partial X}{\partial N_P}$ to denote partial equilibrium results, while $\frac{\mathrm{d}X}{\mathrm{d}N_P} :- \frac{\partial X(N_P, N_F(N_P))}{\partial N_P}$ indicates general equilibrium results, where fringe entry is endogenous.

Profits – partial equilibrium The following two propositions are partial equilibrium results: consider the number of fringe entrants N_F as fixed. The first statement claims that the platform's profits are increasing in its own product variety.

Proposition B.3. *Let Assumptions B.1 to B.3 hold. Also assume that f is continuously differentiable with respect to N_P and also twice differentiable. Let $N_F \geq 0$. Then π_P is also differentiable and*

$$\frac{\partial \pi_P(N_P, N_F)}{\partial N_P} > 0.$$

Proof of Proposition B.3. $f(N_P, N_F)$ is continuously differentiable in N_P, therefore the Leibniz rule can be applied to obtain

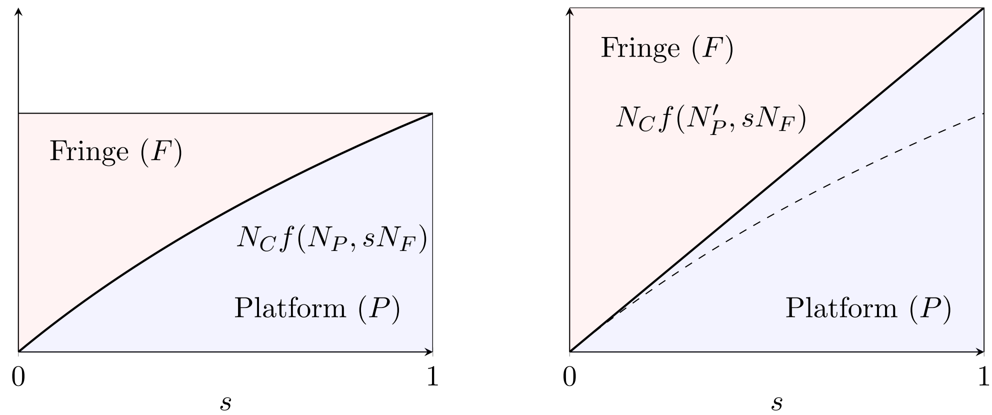
$$\begin{aligned} \frac{\partial \pi_P(N_P, N_F, N_C)}{\partial N_P} &= N_C \frac{\partial}{\partial N_P} \int_0^1 w_P(s) f(N_P, N_F) \mathrm{d}s \\ &= \int_0^1 w_P(s) \underbrace{\frac{\partial f(N_P, sN_F)}{\partial N_P}}_{>0} \mathrm{d}s > 0 \end{aligned}$$

for any non-negative, continuous $w_P(s)$. □

Let us examine what this result does and does not mean. First, remember that f is increasing in both arguments, and N_F is assumed to be fixed for the moment. Therefore, an increase in N_P also increases the size of the pie the participants bargain over. This result states that the slice of the pie the platform gets increases in this case, too. It does not mean, however, that the *relative share* of the pie that the platform gets is also bigger – the increase guaranteed only in absolute terms. For example, it is possible that for the new, higher value of N_P, the complementarities between the fringe firms become stronger, and the platform's bargaining power decreases.[21] Figure B.2 shows an example of this situation.

Next, let us look at an analogous result for the fringe firms. In their case, the direction of the change depends on the complementarities between the platform and the fringe firms.

[21] In fact, one can show that this is the case if the cross-derivatives of f are negative.



(a) Profit shares with N_P

(b) Profit shares with N'_P

Figure B.2. Illustration of Proposition B.3 in the one-sided bargaining case. The right hand side figure shows a world with larger platform product variety ($N_P < N'_P$). Even though the platform's share of the total profits is smaller in relative terms in that case, it is still larger in absolute terms.

Proposition B.4. *Let Assumptions B.1 to B.3 hold. Furthermore, assume that f, w are twice continuously differentiable. Let $N_F > 0$. Then π_F is also differentiable.*

$$\text{If } \frac{\partial^2 f(N_P, n_F)}{\partial n_P \partial n_F} < 0 \;\forall n_F \leq N_F, \text{ then } \frac{\partial \pi_F(N_P, N_F)}{\partial N_P} < 0,$$
$$\text{if } \frac{\partial^2 f(N_P, n_F)}{\partial n_P \partial n_F} > 0 \;\forall n_F \leq N_F, \text{ then } \frac{\partial \pi_F(N_P, N_F)}{\partial N_P} > 0$$

for all $N_P \geq 0$.

Proof of Proposition B.4. Remember that

$$\pi_F(N_P, N_F, N_C) = N_C \int_0^1 w_F(s) N_F \partial_2 f(N_P, sN_F) \mathrm{d}s,$$

By assumption, f is twice continuously differentiable, therefore ∂_2 is also continuously differentiable in N_P. Thus, the Leibniz rule can be applied to obtain

$$\begin{aligned}\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_P} &= \frac{\partial}{\partial N_P} N_C \int_0^1 w_F(s) N_F \partial_2 f(N_P, sN_F) \mathrm{d}s \\ &= N_C \int_0^1 w_F(s) N_F \frac{\partial}{\partial N_P} \partial_2 f(N_P, sN_F) \mathrm{d}s \\ &= N_C \int_0^1 w_F(s) N_F \partial_{12}^2 f(N_P, sN_F) \mathrm{d}s.\end{aligned}$$

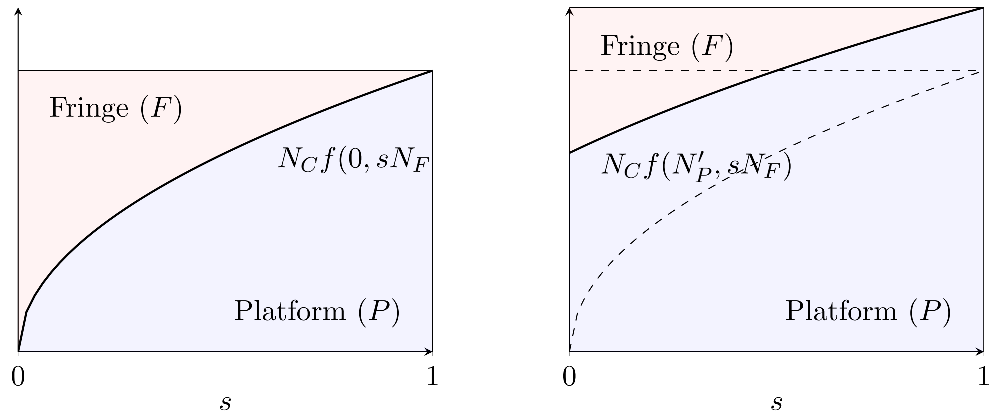
As $w_F(s) \geq 0$ for $s > 0$, if $\partial_2 f(N_P, sN_F)$ has the same sign over $[0, N_F]$, then the integral also has the same sign. Formally,

$$\forall n_F \in [0, N_F] \; \frac{\partial^2 f(N_P, n_F)}{\partial N_P \partial n_F} < 0 \implies \frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_P} < 0$$
$$\forall n_F \in [0, N_F] \; \frac{\partial^2 f(N_P, n_F)}{\partial N_P \partial n_F} > 0 \implies \frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_P} > 0$$

□

In summary, when they are primarily substitutes (the cross-derivatives of f are negative), the fringe's profits decrease as a result of an increase in N_F. The intuition is that, even though the total size of the pie increases, the bargaining power of the fringe deteriorates so much that its total profits decrease not only in relative but also in absolute terms (as illustrated in Figure B.3). On the other hand, when the fringe firms are mostly complements the fringe's profits increase.

As each player's Shapley value is their average marginal contribution to the total value, this result can best be understood through the lens of marginal contributions. When the cross derivative is positive, an increase in the platform's product variety increases the marginal contribution of the fringe firms to the total value for any given number of fringe firms. Therefore, the amount that the fringe gets also increases. On the other hand, when the cross derivative is negative, the marginal contribution of the fringe firms decreases, and so does the amount they obtain.



(a) Profit shares with $N_P = 0$

(b) Profit shares with $N_P = 0.5$

Figure B.3. Illustration of Proposition B.4 when fringe and platform products are substitutes. An increase in the platform's product variety increases total profits, yet, the fringe's share decreases in absolute terms.

Proposition 6 is a direct corollary of the two previous results.

Proof of Proposition 6. The model presented in the main text adheres to Assumptions B.1 to B.3. Thus, by Proposition B.3,

$$\frac{\partial \pi_P(N_P, N_F)}{\partial N_P} > 0.$$

Furthermore,

$$\frac{\partial^2 f(N_P, N_F)}{\partial N_P \partial N_F} = \frac{\partial^2}{\partial N_P \partial N_F} \mu \frac{N_F V_F + N_P V_P}{N_F V_F + N_P V_P + 1} < 0,$$

therefore, by Proposition B.3,

$$\frac{\partial \pi_F(N_P, N_F)}{\partial N_P} < 0.$$

□

Number of entrants – general equilibrium Next, let us turn to equilibrium entry as a function of the platform's product variety. The previous proposition implies an almost immediate corollary regarding the equilibrium number of fringe entrants.

Corollary B.1. *Let Assumptions B.1 to B.5 hold. Furthermore, assume that f, w are twice continuously differentiable. Let N_F^* denote the equilibrium number of fringe firms. Let us also assume that $N_F^* > 0$, and that*

$$\frac{\partial^2 f(n_P, n_F)}{\partial n_P \partial n_F} < 0 \quad \forall n_F \leq N_F$$
$$\frac{\partial^2 f(n_P, n_F)}{\partial n_F^2} < 0 \quad \forall n_F \leq N_F.$$

Then the equilibrium number of fringe firms is also differentiable and

$$\mathrm{d}\partial N_F \mathrm{d}N_P < 0.$$

Proof of Corollary B.1. Proposition 7 establishes that if an equilibrium with $N_F > 0$ exists, it is unique. Use the implicit function theorem on the equation from Assumption B.4 to obtain

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} = \frac{\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_P}}{I_F - \frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F}}$$

The derivative exists if the above expression is well-defined, i.e. $\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F} \neq I_F$. Remember from Proposition B.4 that the numerator of this expression is negative under the condition $\frac{\partial^2 f(N_P, N_F, N_C)}{\partial N_P \partial N_F}$. Now all we need to show to conclude the proof is that

$$\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F} < I_F,$$

which is the statement of Lemma B.2. □

That is, the equilibrium number of entrants increases as a response to an increase in N_P if the platform and the fringe firms are complements and decreases if they are substitutes. The underlying reason is again the concave or hump-shaped fringe profit function. If N_F^* is an equilibrium, then fringe profits minus entry costs are strictly positive for all $N_F \leq N_F^*$

and strictly negative for all $N_F > N_F^*$. Therefore, if an increase in N_P decreases the fringe's profits for every $N_F \geq 0$, equilibrium can be restored by decreasing the number of fringe entrants (and vice versa for the other case). This result is related to the concept of strategic complementary and substitutability, which also depends on the profit function's cross-derivatives (i.e., supermodularity or submodularity).

Based on this general result, the corresponding one in the main text follows immediately.

Proof of Proposition 8. $\pi_F^t(N_P, N_F)$ adheres to the Assumptions B.1 to B.3. Furthermore,

$$\frac{\partial^2 \pi_F^t(N_P, N_F)}{\partial N_P \partial N_F} < 0 \quad \forall\, N_P, N_F \geq 0.$$

Therefore, by Proposition B.4,

$$\frac{\partial \pi_F^t(N_P, N_F)}{\partial N_P} < 0 \quad \forall\, N_P, N_F \geq 0.$$

□

Finally, let us conclude this section by establishing a stronger result for a more restrictive class of profit functions. In the following, I assume that the profit function has an additive form. Intuitively, this means that, in terms of total profits generated, the platform's and the fringe firms' products are substitutable according to some constant ratio.

Assumption B.6. *Assume that f has the following, additive form in N_P and N_F:*

$$f(N_P, N_F) = g(\alpha N_P + \beta N_F)$$

where $\alpha, \beta > 0$, and g is twice differentiable and that $g'' < 0$.

Proposition B.5. *Let Assumptions B.1 to B.6 hold. Then*

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} < -\frac{\alpha}{\beta}.$$

Furthermore,

$$\frac{\mathrm{d}\Pi(N_P, N_F, N_C)}{\mathrm{d}N_P} < 0.$$

Proof of Proposition B.5. Use the implicit function theorem to get the equilibrium number of fringe firms as a function of the platform's product variety:

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} = \frac{\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_P}}{I_F - \frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F}}.$$

Remember, that from Lemma B.2, and the assumption on the investment cost function we have that $\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F} < I_F$. Then, substitute $f(N_P, N_F) = g(\alpha N_P + \beta N_P)$ into π_F to get the following fringe profit function:

$$\pi_F(N_P, N_F, N_C) = \beta N_C N_F \int_0^1 w_F(s) g'(\alpha N_P + s\beta N_F) \mathrm{d}s.$$

Differentiate it with respect to N_P and N_F and substitute into the first expression to obtain

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} = \frac{\alpha\beta N_C N_P \int_0^1 w_F(s) g''(\alpha N_P + s\beta N_F)\mathrm{d}s}{\underbrace{I_F'(N_F) - \beta N_C \int_0^1 w_F(s) g'(\alpha N_P + s\beta N_F)\mathrm{d}s}_{(1)} - \underbrace{\beta^2 N_C N_F \int_0^1 w_F(s) s g''(\alpha N_P + s\beta N_F)\mathrm{d}s}_{(2)}}.$$

From Lemma B.2, we know that the denominator of this expression is positive. Furthermore, the concavity of g implies that expression (2) is also positive. Finally, from Proposition B.4, we have that the whole expression must be negative.

First, let us show that (1) is non-negative, and, coupled with the fact that (2) is positive, omitting it increases the denominator in absolute value, thus making the whole expression larger. To see this, observe that the second part of (1) is just the per-unit fringe profit. Therefore, in equilibrium, (1) must be zero:

$$N_F(1) = N_F I_F - \pi_F(N_P, N_F, N_C) = 0.$$

Next, observe that we can bound (2) from above using the fact that $0 \leq s \leq 1$:

$$(2) \leq \beta^2 N_C N_F \int_0^1 w_F(s) g''(\alpha N_P + s\beta N_F)\mathrm{d}s.$$

For the same reason as before, it also bounds the whole expression from above.

Putting it all together, we have that

$$\frac{\mathrm{d}N_F}{\mathrm{d}N_P} \leq \frac{\alpha\beta N_C N_P \int_0^1 w_F(s) g''(\alpha N_P + s\beta N_F)\mathrm{d}s}{\beta^2 N_C N_F \int_0^1 w_F(s) g''(\alpha N_P + s\beta N_F)\mathrm{d}s} = -\frac{\alpha}{\beta},$$

which is the first statement of the proposition.

To prove the second part of the proposition, differentiate total profits with respect to N_P:

$$\begin{aligned}
\frac{\mathrm{d}\Pi(N_P, N_F, N_C)}{\mathrm{d}N_P}\mathrm{d} &= \frac{\partial \Pi(N_P, N_F(N_P), N_C)}{\partial N_P} \\
&= \frac{\partial}{\partial N_P} N_C g(\alpha N_P + \beta N_F^*(N_P)) \\
&= \underbrace{N_C g'(\alpha N_P + \beta N_F^*(N_P))}_{>0} \left[\alpha + \beta \frac{\partial N_F^*(N_P)}{\partial N_P}\right] \\
&< N_C g'(\alpha N_P + \beta N_F^*(N_P)) \left[\alpha + \beta \left(-\frac{\alpha}{\beta}\right)\right] \\
&= 0.
\end{aligned}$$

□

The first part of this proposition is a stronger version of Corollary B.1. It states that, not only does the equilibrium number of fringe firms decrease as a response to an increase in N_P, but there is a lower bound for this decrease. Moreover, as the second part shows, this bound is sufficient to guarantee that the total size of the pie ($\Pi(N_P, N_F, N_C)$) also decreases in equilibrium.

This result has powerful implications in models where total profits (or total product variety) have a monotone relationship with consumer welfare, such as those in Anderson, Erkal, and Piccinin (2020). The demand system presented in the main text also has this property, and thus, the results of this proposition apply to it, as well.

Proof of Corollary 1. Simply differentiate Equation (7) with respect to N_P. □

Proof of Theorem 3. $\pi^t_F(N_P, N_F)$ adheres to the Assumptions B.1 to B.3, B.5 and B.6. Therefore, by Proposition B.5

$$N_F > 0 \implies \frac{\mathrm{d}N_F}{\mathrm{d}N_P} < -\frac{V_P}{V_F}$$

immediately follows. The second set of results can be obtained from this, as well as Equation (3):

$$\begin{aligned}\frac{\mathrm{d}A}{\mathrm{d}N_P} &= V_P\frac{\mathrm{d}N_P}{\mathrm{d}N_P} + V_F\frac{\mathrm{d}N_F}{\mathrm{d}N_P} \\ &< V_P - V_F\frac{V_P}{V_F} = 0,\end{aligned}$$

where the inequality follows from the previous proposition. Finally, CS is also decreasing as

$$\frac{\mathrm{d}CS}{\mathrm{d}N_P} = \frac{\mathrm{d}\mu\log(A)}{\mathrm{d}N_P} < 0.$$

For the results about the pure retailer regime, use the fact that $A = N_P V_P + 1$ and $CS(A) = \mu\log(A)$. □

Proposition 9 can be viewed as a corollary of the previous theorem.

Proof of Proposition 9. Remember that in the hybrid regime, regardless of N_P, the size o the aggregate is pinned down by the entry fee K_F (Equation (4)). Conversely, when given some A, there is a unique entry fee that supports it:

$$K_F^{impl} = \frac{\mu V_F}{A} - I_F.$$

Furthermore, this entry fee is decreasing in A. Theorem 3 states that A is decreasing in N_P, and thus it follows that

$$\frac{\mathrm{d}K_F^{impl}}{\mathrm{d}N_P} = \underbrace{\frac{\mathrm{d}K_F^{impl}}{\mathrm{d}A}}_{<0}\underbrace{\frac{\mathrm{d}A}{\mathrm{d}N_P}}_{<0} > 0.$$

□

Platform profits – general equilibrium Now, let us consider how the platform's product variety impacts its total profits, while also taking into account its effect on the number of fringe entrants. While Proposition B.3 establishes that the platform's profits increase in its own product variety, it is not a general equilibrium result. In particular, we know that increasing the platform's product variety decreases the number of fringe entrants and total profits. It would be conceivable that this decrease in entry is so large that the platform's profits decrease as well.

While I do not have a general result for the platform's profits in equilibrium, useful observations can be made. No matter the form of the profit function, the platform always prefers having more fringe firms to fewer in the bargaining model. The following proposition formalizes this idea.

Proposition B.6. *Under Assumptions B.2 and B.6, for any $N_C, N_P \leq 0$ and $0 \geq N_F \leq N'_F$*

$$\pi_P(N_P, N_F, N_C) \leq \pi_P(N_P, N'_F, N_C).$$

Proof of Proposition B.6.

$$\begin{aligned}\pi_P(N_P, N_F, N_C) &= N_C \int_0^1 w_P(s) f(N_P, sN_F) \mathrm{d}s \\ &\leq N_C \int_0^1 w_P(s) f(N_P, sN'_F) \mathrm{d}s \\ &= \pi_P(N_P, N'_F, N_C)\end{aligned}$$

with the inequality being strict if $N_F < N'_F$ and f is not constant its second argument on $[0, N'_F]$. □

Proposition 10 is then a special case of this result.

Proof of Proposition 10. Π satisfies Assumptions B.2 and B.6. The result is an immediate corollary of Proposition B.6 with $N_F = 0$ and $N'F = N_F(N_P)$. □

This observation then makes characterizing the profit-maximizing platform product variety in Section 3.4 easier, as one does not have to worry about comparing the different regimes in the case of the bargaining model. If hybrid mode is be feasible for a given N_P, then it is also optimal compared to being a pure retailer. This result allows us to prove an important result from the main text about the platform's profits as a function of its number of products.

Proof of Proposition 12. In the bargaining model, Assumptions B.1 to B.3, B.5 and B.6 are satisfied. From the proof of Equation (B.3), the platform's profits are strictly concave in the entry fee K_F. This implies that for a fixed N_P,

$$\begin{aligned}\frac{\mathrm{d}\pi_P}{\mathrm{d}K_F} &> 0 \text{ if } K_F < K_F^{opt}, \\ \frac{\mathrm{d}\pi_P}{\mathrm{d}K_F} &< 0 \text{ if } K_F > K_F^{opt}.\end{aligned} \tag{B.1}$$

Now let us consider platform profits, but parametrized through N_P and K_F instead of the usual N_P and N_F:

$$\begin{aligned}\frac{\mathrm{d}\pi_P}{\mathrm{d}N_P} &= \frac{\mathrm{d}\pi_P(N_P, K_F(N_P))}{\mathrm{d}N_P} \\ &= \frac{\pi_P(N_P, K_F)}{\partial N_P} + \frac{\pi_P(N_P, K_F)}{\partial K_F}\frac{\partial K_F}{\partial N_P}.\end{aligned}$$

Corollary 1 establishes that

$$\frac{\pi_P(N_P, K_F)}{\partial N_P} = \frac{V_P}{V_F} I_F$$

in the hybrid regime, while Proposition 9 states that

$$\frac{\partial K_F}{\partial N_P}.$$

Together with Equation (B.1), the statement of the proposition follows. □

B.5 Additional lemmas and proofs

This subsection contains a couple of lemmas that, while not very interesting on their own, are necessary for the proofs in the previous sections. Furthermore, it contains the proofs of the results presented in Section 3.2, as they are unrelated to the bargaining framework and are thus omitted from the previous section.

The first lemma helps establish whether Assumption B.5 holds under a specific function f. It states that it is sufficient to show that it holds for some affine transformation of f.

Lemma B.1. *Let $g : \mathbb{R}^+ \to \mathbb{R}$ be a twice continuously differentiable function. Define*

$$G(x) = \int_0^1 g(sx)\mathrm{d}s.$$

Assume that for any $x \geq 0$, $G'(x) < 0$ or $G''(x) < 0$.

Let $h(x) = cg(a + bx)$ for some $a, c > 0$ and $b > 0$. Then the function $H(x) = \int_0^1 h(sx)\mathrm{d}s$ satisfies the same conditions: for any $x \geq 0$, $h'(x) < 0$ or $h''(x) < 0$.

Proof of Lemma B.1. Differentiating H yields

$$\begin{aligned}H'(x) &= \int_0^1 h'(sx)\mathrm{d}s \\ &= \int_0^1 cbg'(a + bsx)\mathrm{d}s \\ &= cbG'(a + bsx).\end{aligned}$$

Similarly, the second derivative is

$$\begin{aligned} H''(x) &= \int_0^1 h''(sx)\mathrm{d}s \\ &= \int_0^1 cb^2 g''(a+bsx)\mathrm{d}s \\ &= cb^2 G''(a+bsx). \end{aligned}$$

Now, for any $x \geq 0$, $y = a + bsx \geq 0$, therefore $G'(y) < 0$ or $G''(y) < 0$. As $c, b > 0$, this implies that $H'(x) < 0$ or $H''(x) < 0$. ☐

The second one is useful for establishing the single crossing property for the fringe profits and investment costs. It states that in an equilibrium, the slope of the fringe profit function is smaller than the investment cost.

Lemma B.2. *Let the conditions in Assumption B.5 hold. Then for any $N_F^* > 0$ for which $\pi_F(N_P, N_F^*, N_C) = I_F N_F$, the partial derivative of fringe profits with respect to the number of fringe firms is smaller than the investment cost I_F:*

$$\left.\frac{\partial \pi_F(N_P, N_F, N_C)}{\partial N_F}\right|_{N_F = N_F^*} < I_F.$$

Proof of Lemma B.2. Assumption B.5 states that at any $N_F \geq 0$, $\pi_F(N_P, N_F, N_C)$ is either concave or decreasing in N_F. In the remainder, let us denote partial derivatives as follows:

$$\partial_F \pi_F(N_P^*, N_F^*, N_C^*) := \left.\frac{\partial \pi_F(N_P, N_F,)}{\partial N_F}\right|_{N_P = N_P*, N_F = N_F^*, N_C = N_C^*}.$$

Notice that the fact that π_F is concave or decreasing in N_F implies that if $\partial_F \pi_F(N_P, N_F', N_C)$ for some $N_F' \geq 0$, then $\partial_F \pi_F(N_P, N_F'', N_C) < \partial_F \pi_F(N_P, N_F', N_C)$ for any $\tilde{N}_F < \bar{N}_F$.

Next, observe that if $\partial_F \pi_F(N_P, 0, N_C) < I_F$, then there is no equilibrium with $N_F > 0$. To show this, assume by contradiction that $\exists N_F^* > 0$ such that $\pi_F(N_P, N_F^*, N_C) = I_F N_F^*$. Then the mean value theorem implies that there is a $\bar{N}_F \in (0, N_F^*)$ such that $\partial_F \pi_F(N_P, \bar{N}_F, N_C) = I_F$. However, this clearly cannot be the case as $\partial_F \pi_F(N_P, N_F, N_C) < I_F$ or $\pi_F(N_P, N_F, N_C) \leq 0$ for all $N_F > 0$.

Now let N_F^* be a positive number for which $f(N_P, N_F^*, N_C) = I_F N_F^*$. It is easy to see that $\partial \pi(N_P, N_F^*, N_C) < I_F$. The reason is that if it exists, then $\partial_F \pi_F(N_P, 0, N_C) > I_F$. Now if $\partial_F \pi_F(N_P, N_F, N_C) \geq I_F \forall N_F > 0$, then $f(N_P, N_F, N_F) > I_F N_F$, and the two functions do not intersect. Therefore, there must exist some $\bar{N}_F > 0$ for which $\partial_F \pi_F(N_P, \bar{N}_F, N_C) < I_F$. This in turn implies that $\partial_F \pi_F(N_P, N_F, N_C) \leq \partial_F \pi_F(N_P, \bar{N}_F, N_C) < I_F$ for all $N_F > \bar{N}_F$. ☐

The remainder of this section contains the proofs related to optimal product pricing and various results for the benchmark model.

Proof of Proposition 3. The profit function for product Ti is the following:

$$\begin{aligned}\pi^v_{T_i}(p_{T_i}) &= (p_{T_i} - c_T)x_{T_i}(p_{Ti}) \\ &= (p_{T_i} - c_T)\frac{\exp\left(\frac{v_T - p_{T_i}}{\mu}\right)}{A}.\end{aligned}$$

Note that A is not influenced by changes in p_{T_i}, as A is an integral and p_T is only changed on a zero-measure (singleton) set. Therefore, let calculate the first order condition while treating A as a constant:

$$\mu\frac{\exp\left(\frac{v_T - p_{T_i}}{\mu}\right)}{A}\left[\frac{p_T - c_T}{\mu} - 1\right] = 0. \tag{B.2}$$

Also note that $\pi_{T_i}(p_{T_i})$ is strictly concave, so the FOC is sufficient for optimality.

Now simply rearrange Equation (B.2) to obtain

$$p^*_{T_i} = c_T + \mu$$

and substitute it into the profit function to get

$$\pi^{v*}_{T_i} = \mu\frac{\exp\left(\frac{v_T - c_T - \mu}{\mu}\right)}{A}.$$

□

Proof of Proposition 4. From Proposition 3, the variable profit of each fringe firm is $\pi^{v*}_{F_i} = \mu V_F/A$. Total profit after entry fees and investment costs is therefore $\pi^{t*}_{F_i} = \mu V_F/A - I_F - K_F$. Under free entry, total profits are zero:

$$0 = \pi^{t*}_{F_i} = \mu V_F/A - K_F - I_F.$$

Simple rearrangement gives the formula we are looking for,

$$A = \mu\frac{V_F}{K_F + I_F},$$

and substituting in $A = N_P V_P + N_F V_F + 1$ yields the equilibrium number of fringe firms,

$$N_F = \frac{\mu}{K_F + I_F} - N_P\frac{V_P}{V_F} - \frac{1}{V_F}.$$

□

Proof of Theorem 1. The total profit function of the platform is the following:

$$\begin{aligned}\pi^t_P &= \pi^v_P + K_F N_F \\ &= \mu\frac{N_P V_P}{K_F + I_F} + K_F\left[\frac{\mu}{K_F + I_F} - N_P\frac{V_P}{V_F} - \frac{1}{V_F}\right].\end{aligned} \tag{B.3}$$

The function is strictly concave in K_F, so the first order condition is sufficient for optimality in the case of an interior solution. Assume that the optimum is indeed interior. Then the FOC is

$$\frac{\mu I_F V_F - (K_F + I_F)^2}{V_F (K_F + I_F)^2} = 0.$$

Rearranging it gives the optimal entry fee

$$K_F^{opt} = \sqrt{\mu I_F V_F} - I_F,$$

and substituting it into the profit function leads to

$$\pi_P^{*t} = \mu - 2\sqrt{\frac{I_F \mu}{V_F}} + \frac{I_F}{V_F}(N_P V_P + 1).$$

Finally, note that

$$\pi_P^{*t} \geq \mu \frac{N_P V_P}{N_P V_P + 1},$$

i.e., the profit that the platform could achieve by excluding the fringe completely. Therefore, the optimum is indeed interior whenever K_F is low enough

Now consider the case when K_F^{opt} is so large that it would lead to no fringe entry. In that case, the platform's only source of profit is selling its own products, and thus

$$\pi_P^t = \pi_P^v = \mu \frac{N_P V_P}{N_P V_P + 1}.$$

□

Proof of Corollary 1. Simply differentiate Equation (7) with respect to N_P. □

Proof of Theorem 2. The first statement follows from differentiating Equation (5) with respect to N_P. $\frac{\mathrm{d}A}{\mathrm{d}N_P} = 0$ follows from the fact that A does not depend on N_P in Equation (4). Finally, $\frac{\mathrm{d}A}{\mathrm{d}N_P} = 0$ follows from the fact that consumer surplus only depends on A.

For the results about the pure retailer regime, use the fact that $A = N_P V_P + 1$ and $CS(A) = \mu \log(A)$. □

Proof of Proposition 11. From Theorems 1 and 2 we have that

$$\frac{\mathrm{d}\pi_P}{\mathrm{d}N_P} = \frac{V_P}{V_F} I_F \text{ if } N_F(N_P) > 0,$$
$$\frac{\mathrm{d}\pi_P}{\mathrm{d}N_P} < \frac{V_P}{V_F} I_F \text{ if } N_F(N_P) = 0.$$

It immediately follows that if the platform can invest in N_P at cost I_P, then the optimal number of products is the one that maximizes profits:

$$\frac{V_P}{I_P} < \frac{V_F}{I_F} \implies N_P^* = 0,$$
$$\frac{V_P}{I_P} > \frac{V_F}{I_F} \implies N_P^* > 0, N_F(N_P^*) = 0,$$
$$\frac{V_P}{I_P} = \frac{V_F}{I_F} \implies N_P^* \in [0, \bar{N}_P] \text{ for some } \bar{N}_P > 0.$$

Figure 7a demonstrates this graphically. □

C Bargaining microfoundations and the cooperative game

This section examines the bargaining assumption from the main text in more detail. I proceed in two steps. First, I provide non-cooperative microfoundations for using the cooperative approach as a reduced-form way of describing bargaining outcomes. Afterwards, I describe the cooperative game in full formality and derive the (weighted) Shapley value of this game.

C.1 Non-cooperative microfoundations for the bargaining outcome

As part of the Nash program[22], there have been several papers proposing microfoundations for the Shapley value in terms of non-cooperative, bargaining-related games (e.g. Gul 1989; Winter 1994; S. Hart and Mas-Colell 1996; Stole and Zwiebel 1996a). The cooperative platform game described in Appendix C.2 satisfies the assumption for many of those models. Any one of those could be used to build microfoundations for the platform game. I will focus on Stole and Zwiebel (1996a) because it specifically pertains to a setting with one indispensable player and many small players, just like the current paper.

The model in Stole and Zwiebel (1996a) is phrased in terms of intra-firm bargaining between the workers and the firm itself. The workers are assumed to have a fixed outside option, and the firm is an indispensable player. This translates directly to the platform game, with the platform being the indispensable player and fringe the firms' outside option having zero value. Furthermore, in Stole and Zwiebel (1996a), the bargaining outcome is interpreted as workers' wages. In the current setting, this translates to bargaining over profit shares. However, as I argue later, it is equivalent to bargaining over entry fees. While this section focuses on the main model with two-sided bargaining and the Shapley value as the solution, the same logic can be applied to the extensions presented in Appendix A, based on sections 3.1 and 3.2 of Stole and Zwiebel (1996a). Finally, the model is defined for a finite number of small players. The continuous version in this paper is the limit of this model as the number of fringe firms goes to infinity.

Let us now describe the bargaining process in the platform game. First, a random order is determined for the fringe firms. This order will remain fixed throughout the bargaining phase.

Then, for each fringe firm (following the order determined earlier), the platform and the fringe firm negotiate over the entry fee. This bilateral negotiation follows the alternating offer

[22] The research agenda aiming to find links between cooperative and non-cooperative game theory, started by nash1953two

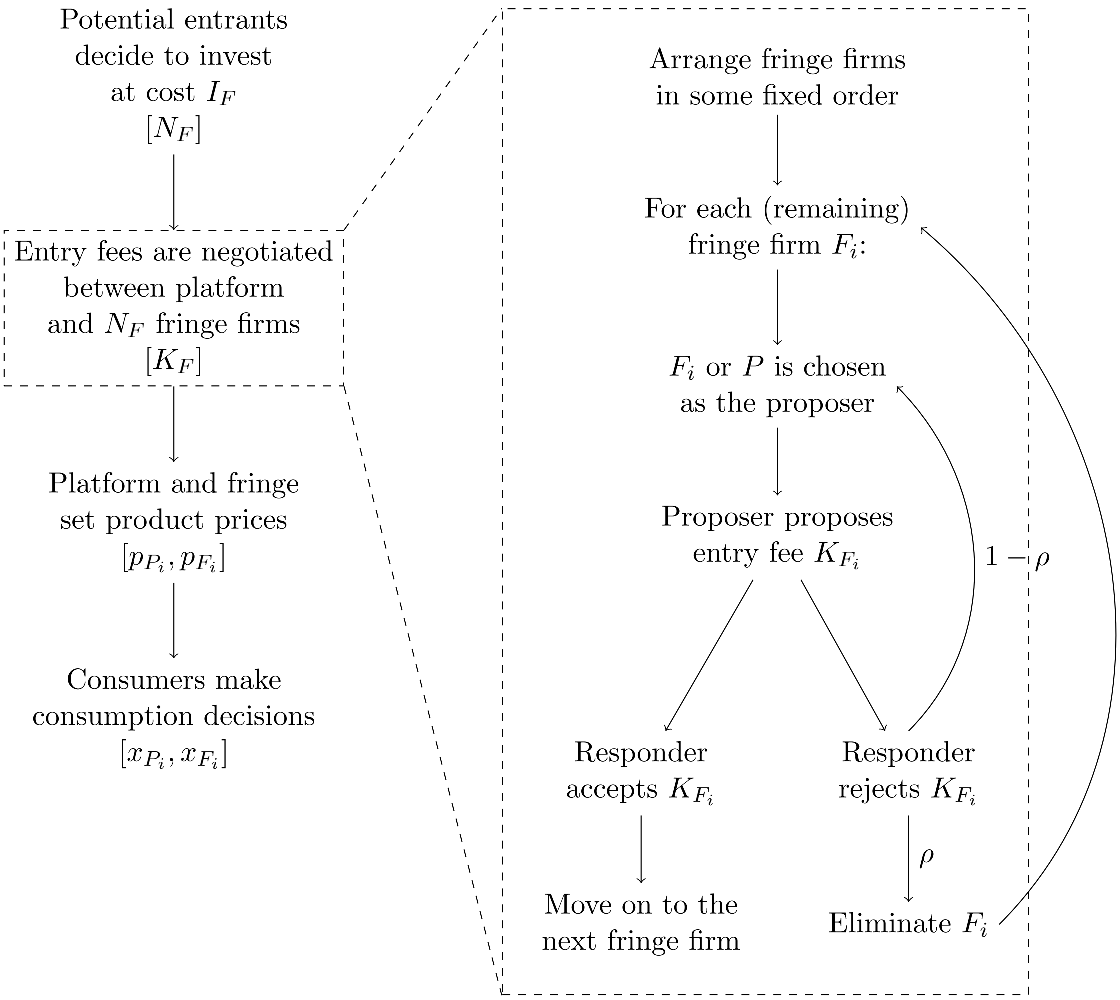


Figure C.1. Timing of the model with extensive-form bargaining. The panel on the right-hand side details the negotiation procedure for determining the entry fees.

procedure described in Binmore, Rubinstein, and Wolinsky (1986). One of the two players is chosen as the proposer and can propose an entry fee for the given firm: K_{F_i}. The other player can either accept or reject the proposal.

If the proposal is accepted, the negotiation moves on to the next fringe firm. If it is rejected, then with probability $1-\rho$, the other player becomes the proposer, and the negotiation continues. Finally, with probability ρ, the negotiations break down, and the fringe firm is eliminated from the rest of the game. Crucially, after a breakdown occurs, all previous agreements are void, and the platform and the remaining fringe firms start the negotiation process from the beginning (following the same pre-determined order, but skipping the eliminated firms).

This bargaining process is repeated until all fringe firms either accept an offer or are eliminated. Figure C.1 illustrates the bargaining process and how it fits into the broader game. Theorem 2 in Stole and Zwiebel (1996a) shows that regardless of the ordering of the fringe firms, the unique subgame perfect equilibrium of this game is the Shapley value of the cooperative game. Note that this result is not only true in expectation: conditioning on any ordering of the fringe firms, the result still holds.

The final piece that is needed to establish the interpretation in terms of entry fees is that for any given configuration of firms emerging from the bargaining process, and, crucially, regardless of the agreed-upon entry fees, the unique equilibrium of the subsequent game is the one described in Section 3.1. According to the previous theorem, players want to agree on a contract such that their final total profits $\pi^t_{F_i}$ are equal to their Shapley value (φ_{F_i}). Given any equilibrium sales profits ($\pi^v_{F_i}$), they can achieve it by setting the entry fees to $K_{F_i} = \varphi_{F_i} - \pi^v_{F_i}$. Finally, Appendix C.2 and Section 3.1 establish that both the Shapley value and the equilibrium sales profits are identical for each fringe firm, and thus a single common entry fee K_F will be agreed upon.

C.2 Cooperative game

Now let us examine how the profit sharing rule from Assumption B.3 can be derived from the random order values of cooperative games. In each of the following subsections, I formally define a cooperative game that models the platform setting and derive the (weighted) Shapley values of the various players. I show that they correspond to the formulas given in Assumption B.3, with a specific choice of weight functions w_P and w_F for each case.

I start with the simplest case: one-sided bargaining and the usual Shapley value (Shapley 1953). Then, I look at weighted values (Weber 1988) in the same cooperative game. Finally, I consider the case when the consumers also participate in the bargaining process, and derive the corresponding Shapley values. Now let us examine how the profit sharing rule from Assumption B.3 can be derived from the random order values of cooperative games. In each of the following subsections, I formally define a cooperative game that models the platform setting, and derive the (weighted) Shapley values of the various players. I show that they correspond to the formulas given in Assumption B.3, with a specific choice of weight functions w_P and w_F for each case.

I start with the simplest case: one-sided bargaining and the usual Shapley value (Shapley 1953). Then, I look at weighted values (Weber 1988) in the same cooperative game. Finally, I

consider the case when the consumers also participate in the bargaining process and derive the corresponding Shapley values.

$$v(S) = \begin{cases} 0 & \text{if } P \notin S \\ f(N_P, n_F(S)) & \text{if } P \in S \end{cases},$$

where $n_F(S)$ is the measure of fringe firms in S.

The following proposition describes the Shapley value of the platform $(\varphi_P(\mathcal{G}))$ and the fringe firms $(\varphi_F(\mathcal{G}))$ in this game.

Proposition C.1. *Consider the cooperative game above. Then, the Shapley values of the platform and the fringe firms are given by*

$$\varphi_P(\mathcal{G}) = \int_0^1 f(N_P, sN_F)\mathrm{d}s,$$
$$\varphi_P(\mathcal{F}) = \int_0^1 sN_F\partial_2 f(N_P, sN_F)\mathrm{d}s.$$

Proof of Proposition C.1. This result is a direct application of Proposition 2 and Corollary 2 in Stancsics (2024). □

As an immediate consequence, if bargaining outcomes are described by the Shapley value, then the resulting allocations satisfy Assumption B.3 with $w_P(s) \equiv 1$ and $w_F(s) = s$.

C.2.1 Weighted value

Now, let us generalize the previous result by assuming that players' shares are described by their weighted values. As before, I start by formally describing the cooperative game. The set of players and the characteristic function are the same as in Appendix C.2, but now the game is additionally endowed with a weight system $\lambda = \{\lambda_P, \lambda_F(i)\}$. Let us assume that all fringe players have the same weight $\lambda_F(i) \equiv 1$. This describes the situation in Appendix A.1.

Then, the weighted values of the players are described by the following proposition.

Proposition C.2. *Consider the cooperative game above. Then, the weighted values of the platform and the fringe firms are given by*

$$\varphi_P(\mathcal{G}) = \int_0^1 \lambda_P s^{\lambda_P - 1} f(N_P, sN_F)\mathrm{d}s,$$
$$\varphi_F(\mathcal{G}) = \int_0^1 s^{\lambda_P} N_F\partial_2 f(N_P, sN_F)\mathrm{d}s.$$

Proof of Proposition C.2. This result is a direct application of Proposition 3 in Stancsics (2024). □

As before, the resulting allocations satisfy Assumption B.3 with $w_P(s) \equiv \lambda_P s^{\lambda_P - 1}$ and $w_F(s) = s^{\lambda_P}$.

C.2.2 Two-sided case

Finally, in certain settings, it might be appropriate to assume that the consumers (or, more generally, the entities on the other side of the market) also engage in the bargaining process. An example for this is Appendix A.2 In such a case, the underlying cooperative game can be formalized as follows.

The set of players consists of the platform plus the fringe firms: $\mathcal{N} = \{P, F_i, C_j\}, i \in [0, N_F], j \in [0, N_C]$. The value of a coalition S is zero without the platform and depends on the number of fringe firms otherwise:

$$v(S) = \begin{cases} 0 & \text{if } P \notin S \\ n_C(S) f(N_P, n_F(S)) & \text{if } P \in S \end{cases},$$

where $n_F(S)$ and $n_C(S)$ is the measure of fringe firms and consumers, respectively, in S.

As in the one-sided case, simple expressions exist for the Shapley value of the platform ($\varphi_P(\mathcal{G})$), fringe firms ($\varphi_F(\mathcal{G})$) and consumers ($\varphi_C(\mathcal{G})$).

Proposition C.3. *Consider the case when only the platform and the fringe firms participate in the bargaining process. Then, the resulting profit shares are given by*

$$\varphi_P(\mathcal{G}) = N_C \int_0^1 s f(N_P, sN_F) \mathrm{d}s,$$
$$\varphi_F(\mathcal{G}) = N_C \int_0^1 s^2 N_F \partial_2 f(N_P, sN_F) \mathrm{d}s$$
$$\varphi_C(\mathcal{G}) = N_C \int_0^1 s f(N_P, sN_F) \mathrm{d}s.$$

proof of Proposition C.3. This result is a direct application of Proposition 7 in Stancsics (2024). □

One thing to note is that the platform's and fringe firms' shares are lower than in the one-sided case. This is due to the fact of needing to share the pie with the consumers, too. Additionally, the platform's share is identical to the consumers' (aggregated) share.[23] This foreshadows the idea that, even though it might not be welfare-maximizing, what is good for the platform might also benefit the consumers in the two-sided case. Finally, as in both examples before, these values satisfy Assumption B.3 with $w_P(s) = s$ and $w_F(s) = s^2$.

[23]This stems from the linearity of the profit function in the number of consumers.